DEMAND AND SUBSTITUTION RELATIONSHIPS FOR FROZEN ORANGE CONCENTRATE

By

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CHAPTER I

THE SETTING FOR THE PROBLEM

Introduction

The Florida citrus industry can be characterized as dynamic in its response to technological developments. Utilization of processing techniques has expanded the number of outlets for citrus products, and new water control and fertilization methods have provided increased potential for production of fruit for these new uses. In the orange segment of the industry both production and utilization of fresh fruit have undergone important changes during the past 15 to 20 years.

Trends in orange production

The quantity of oranges available for consumption in a given year depends on factors ranging from bearing tree acreage, age of the trees, and other physical properties to the nature of the weather throughout the growing season. The interaction of these factors contributes to the annual variability of the crop size. For example, bearing acreage increased from 324,800 to 337,400 acres between the 1951-52 and 1952-53 crop seasons, while orange production declined from 78.6 million to 72.2 million boxes during this same period as a result of declining yields per acre (Table 1). This annual variability tends to disguise the underlying increase in production potential illustrated by the expansion of total and bearing acreage in Florida from 1950-51 through 1963-64. An examination of the trend in production

Table 1.--Florida orange bearing acreage, nonbearing acreage, production, and tree plantings, 1950-51 through 1962-63

Crop season	Bearing acreage	Nonbearing acreage	Total acreage	Percent of total acreage nonbearing	Average yield per acre	Bearing acreage production	Nonbearing acreage production potential	Tree plantings
	000 acres	000 acres	000 acres		Boxes[a]	000 boxes	000 boxes[b]	000 trees
1950-51	309.5	45.3	354.8	12.7	217.4	67,300	10,963	917.2
1951-52	324.8	50.2	375.0	13.4	242.0	78,600	10,743	608.1
1952-53	337.4	49.2	386.6	12.7	214.0	72,200	12,890	698.7
1953-54	348.3	55.3	403.6	13.7	262.0	91,300	13,261	1,102.6
1954-55	368.7	62.7	431.4	14.5	239.8	88,400	14,923	1,445.9
1955-56	382.3	77.7	460.0	16.9	238.0	91,000	18,361	1,566.3
1956-57	393.6	104.3	497.9	20.9	236.3	93,000	23,102	1,882.5
1957-58	372.5	92.8	465.3	19.9	221.5	82,500	21,344	885.5
1958-59	373.9	95.8	469.7	20.4	230.0	86,000	22,388	2,181.9
1959-60	391.6	102.4	494.0	20.7	233.7	91,500	22,313	1,987.2
1960-61	397.8	112.9	510.7	22.1	217.9	86,700	29,783	2,311.1
1961-62	429.8	220.3	650.1	33.9	263.8	113,400	44,258	2,229.5
1962-63	370.7	230.0	600.7	38.3	201.0	74,500	46,230	1,244.2

[a] Boxes containing a net weight of 90 pounds each.

[b] The figures for nonbearing acreage production potential were calculated by multiplying average yield per acre by the respective nonbearing acreage figures.

Source: These data were compiled from Florida Agricultural Statistics Citrus Summary, 1963, Florida Department of Agriculture, Tallahassee, Florida; and Florida Citrus Mutual Annual Statistical Report, 1962-63 Season, Florida Citrus Mutual, Lakeland, Florida.

of oranges over this time span reveals that the orange crop has been increasing in size except for the periods of freeze experienced during 1957-58 and again in 1962-63. Prior to both of these freeze years, the amounts of oranges harvested were the highest recorded over this 14-year period--a 93-million-box harvest during 1956-57 and a 113.4-million-box harvest in 1961-62. Nonbearing acreage, a measure of new groves planted, remained at a relatively constant percentage of total orange acreage from 1950-51 through 1954-55 (Table 1). However, during the next two seasons the amount of total acreage that was nonbearing increased from 14.5 percent to 20.9 percent as a result of tree plantings which exceeded 1.5 million during 1955-56. Moreover, the 1961-62 season ratio of nonbearing acreage to total acreage climbed to 33.9 percent. The apparent profitability in the production of oranges has thus caused a growth in potentially bearing acreage even while actual bearing acreage and the size of the orange crop have been increasing.

The consequences of this growth in potential output are conditioned by the combined effects of an increasing cost situation and a considerable time lag between investment and the realization of returns in orange production. Increased demand for the inputs necessary for orange production has, and will continue to, bid up the price of these productive agents. More output will thus cost more in marginal terms, therefore, implying that eventually a point will be reached where profit will cease to exist for a given level of market consumption and price. Adjustment to and around this point would not be too difficult were it not for the time lag in orange production. A time span of four to six years is necessary for an orange grove to generate a return, and the full potential of the trees is not realized for as much as 10 to 20 years beyond this point. Thus, while current costs and revenues encourage new plantings, the end result of these acreage expansions may drive the price of oranges below the initial planting costs as well as below existing production costs. The problem of overcoming the effects of this lag in production response to expectations is a very important short-run consideration for the individual citrus grower in Florida.

The utilization of the orange crop

In the preceding discussion, the market side of the cost-price adjustment was not considered except to imply that an increased level of supply would tend to result in lower prices to the orange industry. A shift in consumer demand would do much to relieve the pressure of expanded production on the net revenue position of the grower. However, the structure of the consumer market is as complex as the orange production process. Past efforts to develop and expand the retail market for oranges have emphasized new products and advertising. The end result of this emphasis has been the distribution of the fresh orange crop into various forms for retail sale.

The two basic utilizations of orange production in Florida are fresh and processed uses. The processed segment came into being in 1930 with the development and sale of canned single-strength orange juice. The amount of oranges processed for this commodity annually from 1930 through 1935 was very small; however, beginning with the 1935-36 season, the number of boxes of fruit going into processed usage began to steadily increase. Growth of the processing utilization of oranges continued throughout the early 1940's as canned hot-pack concentrate was added to single-strength orange juice as another processed outlet (Fig. 1). While fresh usage levelled off from 1942-43 through 1945-46, the number of boxes of oranges used in processing outlets continued to increase. Frozen orange concentrate, added to the list of processed products, became such a successful outlet for fruit that total processor utilization finally exceeded fresh usage during 1947-48, and with the exception of the following season has maintained this position up to the present time. Moreover, the trend in fresh usage has been downward since 1951-52, resulting in processed utilization being an increasing portion of the expanding supply of oranges experienced from 1952-53 to the current season.

Statement of the Problem

As an agricultural operation, Florida's citrus industry has been interested in obtaining more output for less cost, with technology and productive efficiency being the means of obtaining this goal. Considerable attention has been given to increasing firm profits through cost-decreasing measures, with the revenue side of the economic picture being taken as set by a somewhat complicated and unpredictable consumer market. The expansion in production of oranges has indicated that the productive capability of the entire citrus industry has increased tremendously since 1945-46. However, productive excellence, while still remaining a factor of considerable import to the orange industry, has assumed a secondary role to the attainment of a superior position in the marketplace for Florida's orange products.

Production of frozen concentrated orange juice

Over the period 1953-54 through 1963-64, production and total

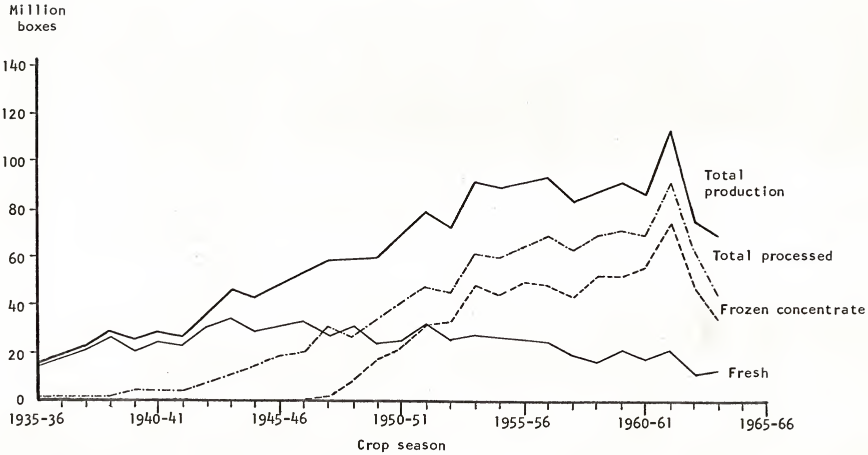

Figure 1.--Florida orange production and utilization, 1935-36 through 1963-64

Source: Florida Agricultural Statistics Citrus Summary, 1964, Florida Department of Agriculture, Tallahassee, Florida.

consumption of concentrate have expanded; however, on an annual basis, greater increases have been experienced in the supply of this product. The inability of the marketplace to absorb all of the available concentrate in one year has resulted from a series of pricing and production practices attributable to the nature of frozen food items. The supply of oranges at the grower level has been increasing annually but has remained relatively inelastic during a given year. The percentage of this expanding production utilized in the manufacture of frozen orange concentrate has increased from 52.8 percent during 1953-54 to a high of 65.0 percent in 1961-62. The supply of concentrate available to consumers, however, has been relatively elastic. The durable character of this commodity has enabled processors to engage in supply control policies designed to head off retail price decreases which could result from an increased amount of orange concentrate being offered to consumers. Thus, the use of carryover stocks as a buffer to support prices has characterized the movement of frozen orange concentrate to the marketplace. However, as production of fresh fruit and concentrate continued to expand, this "buffer" concept, instead of facilitating market adjustments, merely postponed the problems of reckoning with large surpluses of orange concentrate to a future period.

The situation at the end of the 1961-62 season was an example of the end result of accumulating concentrate supplies. Record production levels coupled with expectations of a continuing high output dimmed revenue prospects for 1962-63. Also, a carryover of 33.7 million gallons of frozen orange concentrate, an amount equal to 35.2 percent of orange concentrate consumption that year, still remained in storage (Table 2). The freeze during the early part of 1962-63 served as the

Table 2.--Florida orange production and utilization, 1946-47 through 1963-64

Crop season	Orange production	Oranges processed for concentrate	Frozen orange concentrate pack	Concentrate pack plus carryover	Consumption of frozen orange concentrate	Carry-over of frozen orange concentrate	Carry-over as a percent of consumption
	000 boxes	000 boxes	000 gal.	000 gal.	000 gal.	000 gal.	
1946-47	53,700	466	559	581	431	150	34.8
1947-48	58,400	1,600	1,935	2,085	2,100	750	35.7
1948-49	58,300	8,320	10,232	10,982	8,482	2,500	29.5
1949-50	58,500	17,797	21,647	24,147	18,147	6,000	33.1
1950-51	67,300	22,991	30,758	36,758	29,434	7,324	24.9
1951-52	78,600	31,579	44,031	51,355	43,066	8,289	19.2
1952-53	72,200	32,646	46,554	54,843	50,289	4,554	9.1
1953-54	91,300	48,272	65,531	70,085	58,931	11,154	18.9
1954-55	88,400	44,746	64,877	76,032	67,113	8,919	13.3
1955-56	91,000	49,094	70,224	79,143	66,482	12,948	19.5
1956-57	93,000	48.827	72,012	84,960	74,010	10,950	14.8
1957-58	82,500	43,810	57,185	68,134	61,635	6,499	10.5
1958-59	86,000	52,437	79,911	86,410	70,096	16,314	23.3
1959-60	91,500	51,748	78,149	94,464	84,941	9,523	11.2
1960-61	86,700	55,819	84,298	93,821	80,405	13,416	16.7
1961-62	113,400	73,716	116,082	129,498	95,748	33,749	35.2
1962-63	74,500	47,099	51,648	85,397	69,997	15,399	21.9
1963-64	58,300	34,206	53,674	69,074	--	--	--

Source: Florida Citrus Mutual Annual Statistical Report, 1963-64 Season, Florida Citrus Mutual, Lakeland, Florida.

solution to the production pressure from the fresh fruit component of the industry. However, due to the large carryover from the preceding season, 85.7 million gallons of concentrate were available for consumption. The end of this season saw a carryover figure of 15.3 million gallons of concentrate, or the third largest stockpile of frozen orange concentrate existing at the close of a season since 1945-46. The orange crop for 1964-65 is estimated to be over 80 million boxes, signalling a quick recovery in production levels. This recovery will help to increase the pack of frozen orange concentrate in the immediate future, and recreate the large supply situation that confronted Florida's citrus industry during the 1962-63 season.

Distribution of frozen orange concentrate

The market in which Florida frozen orange concentrate is sold is characterized by active competition and a dynamic structure. A significant feature of the total market for oranges since the inception of frozen orange concentrate has been the levelling of consumption of this frozen product over the last 10 years. On a per capita basis, consumption of concentrate over the period 1953 through 1962 has remained, with two exceptions, between 9 and 12 pounds per year, while purchases of canned single-strength orange juice and fresh oranges have declined (Table 3). In recent years the policy used by the citrus industry has been to differentiate orange concentrate from its competitors in the market on the basis of healthfulness, quality and economy. The ultimate goal of this differentiation, achieved through advertising media, has been to enlarge the concentrate market by making more people aware of the qualities of frozen orange juice compared to other processed juices. Thus, the orange situation is such that attempts

Table 3.--Per capita consumption of frozen orange concentrate, canned single-strength orange juice, and fresh oranges, 1946 through 1963

Year	Frozen orange concentrate	Canned single-strength orange juice	Fresh oranges
		Single-strength pounds[a]	
1946	.08	4.15	14.95
1947	.48	4.11	15.62
1948	1.84	5.03	14.85
1949	3.88	3.87	13.56
1950	6.20	3.37	12.79
1951	8.92	3.81	14.17
1952	10.24	3.58	13.80
1953	11.80	3.13	13.78
1954	13.24	3.08	12.10
1955	12.88	2.95	12.74
1956	14.08	2.42	11.45
1957	11.52	2.45	10.42
1958	12.88	2.66	8.82
1959	15.32	1.91	10.16
1960	14.28	2.13	9.98
1961	16.56	1.71	8.02
1962	12.00	1.93	6.74
1963	9.44[b]	--	--

[a] The consumption figures for frozen orange concentrate were converted to a single-strength basis by a 4 x 1 multiplication. Fresh consumption was converted to a single-strength basis based on annual yields of gallons of juice per 90-pound box.

[b] Preliminary.

Source: This information was compiled from Florida Citrus Mutual Annual Statistical Report, 1962-63 Season, Florida Citrus Mutual, Lakeland, Florida; and Per Capita Consumption of Citrus Products, Florida Citrus Commission, Lakeland, Florida, September 30, 1963.

to increase demand are being made. However, the pressure of increased supplies of concentrate is fast becoming a reality with which advertising alone may not be able to cope in the future.

In order to formulate a complete marketing strategy for frozen concentrated orange juice, Florida's citrus industry must supplement its product differentiation procedures with tactics relevant to various market segments. Within the market for orange concentrate, a physically homogeneous product group, there exists three distinct segments. These divisions, characterized by differences in advertising, merchandising and pricing techniques, are classified as nationally advertised labels, packer labels and private labels. In light of the fact that frozen concentrated orange juice will play an important role as a major outlet for Florida's expanding orange production, the citrus industry needs to be cognizant of, and respond to, the nature of consumer demand which has supported this segmentation. The final product of this awareness should then be the formulation of a general marketing policy in preparation for future supply conditions greater than are currently in evidence.

Objectives of This Study

Management decisions at any level of business operation should be made through a combination of analysis of facts and rigorous reasoning. In specifying a marketing policy, one of the most difficult problems is the assimilation of information concerning consumer reaction to the various alternative strategies under consideration. Promotional devices, product development and pricing actions cannot be adequately investigated without knowledge of the consumer demand for the commodity in question.

Total demand

The first objective of this study was to estimate the price elasticity of demand for frozen orange concentrate in terms of three brand segments and a composite demand curve. Hence, these segments, hereafter referred to as the national label, the packer label and the private label, were analyzed as functions of their respective prices. The structure of the total demand function was then investigated as a combination of components within the product category of concentrate.

Substitution relationships

The existence of three segments within the market for frozen orange concentrate differentiated essentially by label and price characteristics implies that consumers could be shifting their purchases among these brand-types as prices change. Thus, the prices of the various brands could affect the sales of all brands in some manner if economic substitution existed among the three segments of the market. Therefore, the second objective of this study was to ascertain if economic substitution did occur within the three segments of the frozen orange concentrate market. This was accomplished through an estimation of the price-cross elasticities of demand among national, packer and private labels.

These estimates, coupled with the direct price effects in the first objective, will contribute demand information needed by management in formulating future marketing strategies for frozen orange concentrate.

Methodology for Providing the Information

The culmination of the efforts of production and marketing processes is found in the acts of retail sales operations. By exercising the right of purchase, the consumer either approves or disapproves of the conditions resulting from these economic processes, and the processes are perpetuated or altered, whichever the case may be.

The various forms of utility created by producing and marketing agencies are therefore of little consequence to anyone unless the consumer of the products embodying this utility will in fact purchase the item in question. Consumer demand indicates the quantities of a product that will be purchased at varying prices for a given market at a given time. There are several research techniques that have been utilized in attempting to analyze these sales acts and estimate price elasticity of demand for them.

Consumer preference surveys

Retail market activity for a product can be described by observing buyers in the act of shopping for the product. This observation can be either on-the-spot recordings of shopping habits or impersonal summing of store sales for some time period. In either case, no conditions are controlled; the process is descriptive of a market within which all factors influencing the decision to buy are variable. The analysis of this type of information for the effect of a single factor is, in most cases, impossible.

If a consumer-household survey is conducted at the same time and in the same locale as a general observation study, the two sets of data can be combined in a complementary manner. The characteristics of the population observed can be ascertained in the survey, and the questions asked can be designed to check on consumer attitudes about a product by various classifications. The comparison of observed buying with stated preferences helps to test the validity of both systems of market description.

While observation and surveys can provide insight into the total nature of a market, neither method allows the researcher to isolate and analyze consumer buying activity with respect to a single factor. Specifically, estimation of the price elasticity and cross elasticity of demand for more than one brand of frozen orange concentrate would be extremely difficult without some controls being exerted in the market observed.

Consumer panels

A method of obtaining precise records of consumer purchases over a long time period is through the use of a consumer panel. Usually a panel consists of a sample of families providing a cross-sectional structure over specified family characteristics. The characteristics might be income, education and size of family. The cooperating family units maintain records of their purchases of selected food items, the prices paid for these items, and usually information concerning where the items were purchased.

The consumer panel is a useful technique in that it provides very accurate information about consumer expenditures. Furthermore, the data can be examined under various family-characteristic breakdowns and over specified time intervals. The effects of structural changes or technological innovations in a particular industry can also be observed. In new product research, test marketing results obtained through a panel can be examined to determine acceptability of the product and possible needs for new development. Thus, the numerous ways in which panel data can be used make it a sound method for describing the consumer side of a market. This description, however, is one within which all the economic and social factors are variable. A method of reducing some of this variation is to investigate consumer activity over one particular group of family characteristics. If the analysis is over a single group by classification, then the accuracy of the data and the precision with which the population can be subdivided are advantages which make panels an ideal source of information for time series price analysis.

Time series analysis

An assumption underlying the use of time series analysis for economic forecasting and description is that the present and the future are to some extent conditioned by and patterned after the past. The study of relationships over time, however, is both an advantage and a problem in investigating the demand for a product.

The completeness and accuracy of properly collected and classified historical data cannot be challenged. Using regression techniques and equipped with a priori knowledge as to what to look for, the trained researcher can describe with precision and confidence the relationship between specified variables. The data can be adjusted for disturbing influences, if these influences can be identified and their effect measured and can be used to show the nature and magnitude of structural changes in an industry or market over time. The problem of all components being variables still exists though, and is even further complicated by this analysis over time. Factors influencing consumer expenditures influence each other, and in some way affect the consumer expenditures themselves. The pluralistic and complex nature of economic data thus makes it very difficult to obtain price elasticity and cross elasticity estimates of demand free from the effects of other economic forces.

The controlled pricing technique

A method of data generation for demand analysis that attempts to overcome the major shortcomings of information collected by uncontrolled observation is that of controlled pricing. First, the multitude of factors affecting consumer purchases of a product are either assumed to be or are by actual physical action held constant for the duration of the study. Second, the price of the commodity concerned is varied according to a predetermined statistical design in an effort to generate data from which estimates of the demand parameters consistent with an established economic model can be made. Price variability can also be induced for specified competing products in the event that substitution effects are of interest.

In regard to exogenous factors such as consumer incomes, tastes and preferences, and prices of other goods, the duration of a controlled pricing experiment is assumed to be short enough that these factors will not change sufficiently to influence the study results. Promotional policies related to the merchandising of the commodity studied are physically held constant. Thus, the reaction of consumers to price changes can be evaluated over a determined range of prices within a short period of time and under a fixed market situation. This does not mean that the effects of the exogenous factors are removed, rather that they exist, but are constant, thus allowing the researcher to test the direction and magnitude of variation in consumer purchases associated with price changes. If more than one product is studied, the amount of substitution between the two commodities based on price variation of both can also be analyzed.

Review of the Controlled Pricing Technique

Previous utilization

The atmosphere of a laboratory has always been an attribute enjoyed by the physical scientists and envied by the social scientists. The multiplicity and plurality of factors influencing human behavior have always effectively blocked the implementation of completely isolated experiments striving to precisely describe such principles as consumer demand. While complete regulation and specification of a marketing environment has been impossible, efforts have been made to analyze the major factors influencing consumer purchases under highly controlled circumstances.

The general technique of controlled pricing was first used to estimate demand for oranges by Godwin during 1952.[1] The objectives of this study were:

(1) Determination of the nature of demand for fresh oranges in a limited area.

(2) Investigation of the feasibility of using experimental methods in estimating the demand for citrus products.

The experimental units were seven retail supermarkets in central Kentucky. A seven-by-seven latin square design utilizing stores and weeks as column and row variables was the basic system around which the project evolved. The segment of the demand for fresh oranges which was estimated was defined over a range of prices per dozen oranges divided into 5-cent differentials. The resultant treatments, stated as positive or negative deviations from the mean price or established market price, were allocated to the experimental units subject to the restrictions associated with a standard latin square design. The total design provided a system which exposed customers shopping in the seven stores over the 7-week period to all price levels over both weeks and stores. The manipulation of prices was carried on with an atmosphere of controls which covered quality of the fruit, display size and general management of merchandising techniques normally used for fresh oranges in the stores.

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[1] M. R. Godwin, Customer Response to Varying Prices for Florida Oranges (Gainesville, Florida: Florida Agricultural Experiment Stations Bulletin 508, December 1952).

Sales of fresh oranges were recorded in conjunction with the varying price levels and were expressed in terms of pounds sold per 100 customers in each experimental unit. The effect of price on sales was tested through standard analysis of variance procedures associated with factorial experiments, and was found to be statistically different from zero. The demand equation which resulted from the analysis was expressed as:

$$Y = aX + bX^2 + c \tag{1.1}$$

where:

Y = quantity of fresh oranges sold in pounds per 100 customers

X = the price charged per dozen for fresh oranges

a = regression coefficient estimating the linear effect of price on sales

b = regression coefficient estimating the quadratic effect of price on sales

c = regression constant associated with the sales of fresh oranges

Calculation of the regression coefficients resulted in the following equation:

$$Y = -2.114X + .0764X^2 + 54.098 \tag{1.2}$$

A standard arc elasticity formula was utilized in calculating coefficients of price elasticity along the demand curve approximated by this equation. The over-all elasticity for the curve was found to be -1.160, with the curve remaining elastic as prices were lowered, and becoming slightly inelastic as prices were increased relative to the prevailing market price. The highest elasticity was -1.516 and occurred in the region of the market price. As prices were decreased, the degree of elasticity diminished but remained greater than unity. Increases in price above the market level resulted in the elasticity of demand decreasing and eventually becoming less than unity. The success of this project in terms of operation and estimation procedures was of considerable aid in the development of a similar study concerned with frozen orange concentrate.

During the summer of 1954, Godwin and Powell[2] conducted this subsequent experiment in which customers were exposed to varying price situations for frozen orange concentrate. Ten retail supermarkets in the vicinity of Trenton, New Jersey were used as test stores over a period of nine weeks. While the basic concept of controlled pricing was in concurrence with the procedure used in estimating demand for fresh oranges, the structure of the objectives and ensuing analysis was expanded considerably. Interest was not only centered on the demand curve for orange concentrate, but also on the possibility of short-run shifts in both level and slope of this function over the duration of the experiment. Consequently, the data generation procedure was structured to allow investigation of a price age or price carryover effect from week to week. Realization that a nationally advertised brand, packer brand and private brand were being sold in the store was accounted for by maintaining the same price differential between these brands for any given price treatment. However, no provision was made for investigating the degree of economic substitution that might occur among the brand-classes.

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[2] M. R. Godwin and L. A. Powell, Sr., Consumer Reaction to Varying Prices for Frozen Orange Concentrate (Gainesville, Florida: Florida Agricultural Experiment Stations Bulletin 589, August 1957).

Analysis of the sales figures collected in conjunction with the various price conditions revealed that consumers were resistant to price increases and increasingly sensitive to price decreases. A logarithmic function, fit to a price variable taking on the values of 8.5, 10.5, 13.5, 16.5 (market price during the study), and 20.5 cents per 6-ounce can, gave an elasticity of unity at the 13.5 value. As prices were increased to 16.5 and 20.5 cents, price elasticity of demand became more inelastic; while at the lower price ranges, elasticity values became more elastic. The effect of a price age on elasticities could not be ascertained statistically. However, a shift in sales associated with weeks within a given price age led to the suggestion that consumers actually did go through a period of adjustment to a new price situation over time.

A controlled pricing experiment to investigate both demand and substitution relationships for fresh oranges was used by Chapman during the spring of 1962.[3] The objectives of this study were:

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[3] W. F. Chapman, Jr., Demand and Substitution Relationships for Florida and California Valencia Oranges Produced for Fresh Market (Ph.D. dissertation, University of Florida, 1963).

(1) Estimate the price elasticity of demand coefficients for Florida Indian River, Florida Interior and California oranges.

(2) Estimate cross-price elasticity of demand coefficients for Florida Indian River, Florida Interior and California oranges with respect to the prices of the other two oranges.

The economic model which was established provided that within the realm of a given market the sales of each orange classification were primarily a direct function of the price of each respective type and an indirect function of the price of the remaining two varieties. Thus, a system of equations of the following form were established:

$$Q_1 = f(P_1, P_2, P_3 | Z) \quad (1.3)$$

$$Q_2 = f(P_1, P_2, P_3 | Z) \quad (1.4)$$

$$Q_3 = f(P_1, P_2, P_3 | Z) \quad (1.5)$$

where:

Q_1, Q_2, Q_3 = the quantities sold of the different varieties of oranges

P_1, P_2, P_3 = the respective prices of the three orange classifications

Z = demand-influencing factors other than price which were assumed to be constant during the conduct of the experiment

A series of varying combinations of prices for the three orange categories was selected from an over-all price range relevant to the time and area within which the study was conducted. These combinations provided a set of price situations over which both demand and substitution relationships could be investigated. Estimates of the various demand parameters were made using data generated under these price treatments in a closely regulated retail supermarket atmosphere. This study marked the first successful attempt to measure cross-price elasticity coefficients for oranges using the controlled pricing technique.

There has been a considerable amount of work done in the area of demand estimation for a large and varied number of commodities. Recognition of more work has not been accorded in the previous discussion, primarily because the general philosophies as well as specific procedures used in this study were based on the three projects reviewed. The general concepts of controlling a retail environment with respect to nonprice factors were developed from procedures employed in the first study by Godwin. In addition, transformations performed on the dependent variable and the ensuing statistical analysis were established after consideration of the techniques utilized in the first study conducted by Godwin, as well as the experiment concerning frozen orange concentrate carried out by Godwin and Powell. Finally, the basic structure of the demand functions and experimental design utilized by Chapman provided the foundation for estimating substitution relationships to satisfy the second objective of this study.

The importance of control

From a practical viewpoint, investigation of consumer demand in a framework within which all of the factors operating in the marketplace were quantified and explained would be difficult, if not impossible. The classical definition of a demand curve in theory collects these factors into major classifications and then postulates as to their relative effects given that certain variables are held constant. Thus, investigation of the relationship between quantity sold and price for any commodity should be within the framework of assumptions which support the economic theories of consumer demand. Such was the case in each of the studies previously mentioned. All merchandising practices and promotion campaigns for the products involved were controlled during the conduct of each study. This procedure limited the number of major exogenous influences that could have affected customer buying decisions while prices were being altered.

Another reason for control in the process of generating economic data relates to the area of econometrics and the use of stochastic economic models. The large degree of variability in economic data is the reason for assuming that the error term in any econometric model is stochastic. There are four reasons for the divergence in observed data from what may be hypothetically expected.[4] First, the model may not be complete in that relevant variables or equations could be excluded from the model's structure. Second, specification of the mathematical form of the model may be incorrect. The third reason is that economic aggregates are totalled over a large number of different individuals who act independently of one another. Last, there may be errors of measurement in the data.

The controlled pricing experiment, if carefully conducted, provides data within which the last item mentioned above is minimized. The short time span and area restrictions usually incorporated in this research technique also help to limit the possibility of excluding relevant variables from either control by design or analysis. The effects of exogenous factors other than price are either held constant or influence demand so little in the short-run that the data generation process is not affected. Moreover, the specification of the exact treatments that will be utilized not only establishes the range of the price variable with certainty, but allows estimation of cross-price elasticities among products. The manipulation of price in a demand study is by far the most important aspect of a controlled pricing experiment. This is not to say that regulation of exogenous forces is not critical to the entire procedure, but rather that alteration of prices within a regulated merchandising atmosphere is the important factor allowing collection of useful data for analysis.

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[4] S. Valavanis, Econometrics (New York: McGraw-Hill Book Co., Inc., 1959), p. 5.

Specification of the Research Methodology and Its Limitations

The suitability of the controlled pricing technique to problems involving the simultaneous determination of the effects of several variables on one dependent variable enhanced its applicability for use in this study. Specification of an economic model similar to Chapman's involving three products and their respective prices, coupled with data generated in controlled retail supermarkets over a varying price system, was envisioned as providing a structure within which an analysis of the demand and substitution relationships for frozen orange concentrate could be carried out.

Since it was the objective of this project to estimate demand and substitution relationships as a function of price variation, the controlled pricing technique comprised the basic foundation for the research methodology.

The main disadvantages of using this technique relate to cost and the degree of generality of the results. There can be little question that the cost of manipulating an existing market situation is expensive. Personnel requirements for planning, administration and operation are of particular concern in that the results can be only as accurate as the data collected. In addition, some expense incurred by the cooperating firm due to the conduct of the project must, in all fairness, be borne by the research agency. Finally, the nature of the product under study will dictate the extent and cost of special operational procedures that must be implemented beyond the normal activity in a retail grocery store.

The question of lack of generality in the findings is of particular concern. The calculated coefficients are estimates of price elasticity of demand over a given price range, in a given area and during a given time period for a particular study. The validity of such coefficients is a function of the data collected and the statistical error involved. Any attempt to generalize beyond the price and area specifications of any study without perfect knowledge of the population variance and the effects of factors previously assumed to be exogenous would be conceptually impossible. Although the possibility exists that these estimates are close to the true population parameters, the current state of research technology does not allow verification of this hypothesis. In the same manner, estimates of over-all demand parameters based on national averages cannot be applied to a given region or time period. Thus, the question resolves into one of determining for a stated objective the best estimates of demand parameters for a given situation. It was felt in this study, for reasons cited earlier, that the controlled pricing technique provided the most suitable basis for developing analytical procedures that would give efficient estimates of the demand and substitution relationships for frozen orange concentrate.

CHAPTER II

DEVELOPMENT OF THE ECONOMIC MODEL

The critical assumption in the specification of any model approximating human behavior is that there is some order in the total environment encompassing this behavior and that this order can be ascertained. The formulation of theories concerning behavioral phenomenon allows the researcher to postulate the structure of the situation under consideration, eventually developing an approximate model which is a subset of these theories. In terms of demand studies, this process encompasses the conceptualization of the socioeconomic relationships involved and the specification of an economic model which can be logically evaluated through observation.

Theoretical Framework

Utility and demand

The consumer can be regarded as an economic unit, purchasing goods and services for some use, and producing satisfaction or utility by consuming the purchased items.[1] Thus, the satisfaction of desires can be construed as the production of utility by an individual. A parallelism can be drawn between the demand of a consumer for a product and the demand of a firm for a factor of production given the above assertions and the following assumptions:

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[1] K. E. Boulding, Economic Analysis (Third ed.; New York: Harper & Brothers, 1955), p. 680.

(1) The utility an individual produces while consuming a good or service, while unmeasurable in quantitative terms, can be depicted conceptually.

(2) All goods and services are regarded as consumption inputs, and can be held constant in some optimum proportion while the utility structure of a single input is being analyzed.

(3) Time is fixed at one point.

If the marginal rate of substitution of money for consumption inputs is defined as the amount of money that holds the same utility as one unit of a consumption input, then a curve representing this relationship will constitute a schedule of quantities of a given item that will be demanded for consumption at a given set of prices.[2] The slope of this curve will be negative due to the validity of the law of diminishing marginal utility. If the major concern in this study was the demand for one product, conceptualization of the economic model could proceed from this point. However, more than one commodity was investigated, thus calling for further discussion of the general principles of demand theory.

The indifference concept

The indifference curve approach to demand analysis requires that the second assumption above be slightly revised to include investigation of two products with all other items held constant in some fixed proportion. The consumer is now faced with a series of combinations of the two goods which are assumed to produce the same total utility. The slope of an indifference curve approximating these combinations in a single utility plane is the marginal rate of substitution between the two products. Construction of similar curves in different utility planes will result in an approximation of the total utility surface for these inputs. Differentiation of the function representing this surface with respect to one of the products, followed by conversion to the marginal rate of substitution of money for this product, will result in derivation of the consumer's demand for this item. The nature of this demand curve is conditioned by the level at which the other product is assumed to be constant, as well as the variability in the marginal rate of substitution of money for the other product at varying levels. Because all of these factors are utilized in deriving the demand curve for each product separately, there is an inherent relation between the demands for both products. It follows that these assertions could be extended to any number of products with the major complication being the addition of dimensions to the total utility surface of the individual.

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[2] *Ibid.*, p. 685.

In this study, three classes of brands were defined within the category of frozen orange concentrate. It was assumed that the consumer considered an array of products in making a buying decision with respect to one item. In that this consideration involved prices as well as given attitudes and preferences, an analysis of the final decision to buy was made in terms of a general theory of differentiation and related demands.

There are two categories of product differentiation. The first classification, which will not be of concern in this study, is the area of physical differences among products destined for the same end use or different end uses. The second area is concerned with products destined for the same end use which are physically similar, but differentiated by trademarks or labels. This second category implies an attempt to stimulate the consumer's mind with respect to competing products through the use of various forms of sales promotion, particularly those of advertising. The measure of the success of promotional activities is found in consumer purchases; however, the nature of the consumer's attitude is not the only factor influencing quantities sold for consumption. The introduction of the marketplace brings the roles of price and promotion together, and demand analysis necessarily takes into account the influence of both factors on consumer behavior.

Consumer behavior.--Human or consumer behavior is grouped into three components: motivation, cognition and learning.[3] Motivation embodies a disequilibrium of human needs that creates tensions which the consumer seeks to relieve. In the process of seeking relief, the consumer will experience a drive or force causing him to take some sort of action to achieve a goal that possesses factors capable of relieving these tensions. Thus, motivation initiates behavior.

Cognition refers to the mental operations which serve as guiding factors in determining the direction and action taken by a consumer in seeking to satisfy a need. These cognitive processes could be thought of as responses to motivations which directly condition behavior. Moreover, this condition of behavior not only depends on the individual's psychological makeup, but on the current environment surrounding him, and the consumer's expectation of the utility producing power of a consumption input. Within this cognitive structure there exists generic expectancies embodying ideas surrounding a class of products which compete for the same use and object expectancies referring to the utility power of items within this class.[4] The character of these expectations depends on a number of factors encompassing past exposure to the commodities in question. In the framework of this study, a consumer's generic expectancy refers to certain ideas possessed about frozen orange juice, while object expectancies are peculiar to characteristics for given brands of concentrated orange juice. The nature of differences in an individual's mind is not important here. The prime concern is the fact that these differences exist and are manifested in the purchases of different labels of orange concentrate.

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[3] J. A. Bayton, "Motivation, Cognition, Learning--Basic Factors in Consumer Behavior," Managerial Marketing: Perspectives and Viewpoints, ed. W. Lazer and E. J. Kelly (rev. ed.; Homewood, Ill.: Richard D. Irwin, Inc., 1962).

A partial conditioner of consumer object expectancies is the process of learning. This deals with the results of the consumption operation. After a person is stimulated to respond to a drive and takes action through some cognitive system, he consumes the good and derives the satisfaction of his original need. This satisfaction is remembered when the drive reoccurs, and conditions the amount of cognitive activity that will ensue in the behavior process. Therefore, the consumer learns about the utility of an item; and if his expectations are satisfied, lapses into a habit routine in which his need-satisfaction for that good requires very little cognitive activity.

Motivation is the least important of the three factors of consumer behavior in analyzing product differentiation. The reason for this is that a need is felt by the consumer for some satisfaction which can be fulfilled by a number of competing products. Therefore, the cognitive and learning processes are the most important in that they involve knowledge of the product's varying characteristics. Under these assumptions, the stimulus to product differentiation is initiated at the learning level and conditioned at the cognitive level. The consumer's response to differentiation is found only at the cognitive level.

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[4] _Ibid_., p. 628.

The first-order stimulus towards differentiation of products under label and trademark competition is that of advertising. Advertising of a new product, or a product not familiar to the consumer, exposes him to its characteristics without his actually consuming it. The extent to which the consumer remembers these characteristics influences his cognitive awareness of the product and is in turn conditioned by his environment. When the consumer is faced with a cognitive decision concerning the generic class of which this product is a member, the strength of this advertising as remembered by him results in some purchasing action as a response to differentiation. Once the product is consumed, the differentiation becomes supported by the individual's actual learning process, causing him to generate his own pattern of differentiation of the good with respect to other goods. This utility pattern will remain steady until another stimulus is introduced to his system.

Advertising is not the only stimulus for product differentiation by a consumer. A price change among the competing products can also affect the utility surface of a generic class of goods. The nature of the competition between products can thus be stimulated in a number of ways embodying combinations of various types of differentiating procedures, with price considerations interacting with the consumer's attitude at the time of sale. Thus, the consumer approached the act of purchasing one of the three brands of frozen orange concentrate used in this study with a given behavior pattern commensurate with the environment to which he was accustomed. Within this behavior pattern, the three brands were interrelated in that the consumer was aware of them and had established a particular classification of each brand with respect to its utility-producing power for him.

The existence of product differentiation in the consumer's mind was regarded as a function of behavior patterns which were activated and conditioned by psychological and economic factors. With respect to the three classes of brands of frozen orange concentrate used in this study, it was assumed that the psychological stimuli on the cognitive processes of consumption remained fixed. The initiating force for rearranging the consumer's utility surface for the three brands was price alterations, with any change in object expectancies being a function of price-induced consumption. In this manner, the interpretation of the degrees of differentiation and rates of substitution among the brands was carried on within the framework of economic theory.

The Economic Model

Within the product category of frozen orange concentrate, there are many brands differentiated by degrees of promotion and pricing policies. These two policies operate together to maintain a retail market situation in which nationally advertised brands of concentrate command the highest prices while packer and private labels sell at comparatively lower price levels. The evaluation of the consumer's cognitive approach to changing price situations for various brand-types of concentrate provided the guidelines for establishment of the economic model. The total quantity of concentrate sold as a function of individual brand prices plus the relationship among brands at different price levels was expressed as:

$$Y_i = F_i(X_1, X_2, X_3 \ . \ . \ . \ X_n) \qquad (2.1)$$

where total quantity taken, Y_i, is a function, F_i, of the prices, X_n, and the "n" items offered. There are other influences besides price operating on Y_i; however, one of the main assumptions underlying the controlled pricing technique used in this study was that the effects of these nonprice variables were constant or controlled by design.

In this study, the total quantity, Y_i, was subdivided into three components, Y_1, Y_2, Y_3, which represented the sales of the three brands used. This procedure gave the following set of equations:

$$Y_1 = F_1(X_1, X_2, X_3 | T, Z) \qquad (2.2)$$

$$Y_2 = F_2(X_1, X_2, X_3 | T, Z) \qquad (2.3)$$

$$Y_3 = F_3(X_1, X_2, X_3 | T, Z) \qquad (2.4)$$

If no prior knowledge of the product relationships was available, the nature of the cross elasticity of demand between the brands could indicate complementary, independent or competitive relationships. Defining:

X_i = quantity of item i

X_j = quantity of item j

U = total utility of an individual

$\frac{\partial U}{\partial X_i}$ = partial derivative of U with respect to item i

$\frac{\partial^2 U}{\partial X_j \partial X_i}$ = the second partial derivative of U with respect to item i and item j

Then, item i and item j can be classified in the following manner:

Competing: $\frac{\partial^2 U}{\partial X_j \partial X_i} < 0$ (2.5).

Independent $\frac{\partial^2 U}{\partial X_j \partial X_i} = 0$ (2.6)

Complementary: $\frac{\partial^2 U}{\partial X_j \partial X_i} > 0$ (2.7)

However, the above classification is in terms of the marginal utility of the goods. In order to obtain the same type of identification in terms of price and quantity, the following definitions must be construed:

Y = money

$\frac{\partial U}{\partial Y}$ = marginal utility of money expenditure

$X_i \cdot \frac{\partial U}{\partial Y}$ = X_i in terms of the marginal utility of money[5]

If $X_i \cdot \frac{\partial U}{\partial Y}$ is differentiated with respect to X_j, the following result is obtained:

$$\frac{\partial X_i \cdot \frac{\partial U}{\partial Y}}{\partial X_j} = \frac{\partial U}{\partial Y} \cdot \frac{\partial X_i}{\partial X_j} + X_i \cdot \frac{\partial^2 U}{\partial X_j \partial Y} \qquad (2.8)$$

Assuming constant marginal utility of money, the second term in the above derivative becomes zero and the goods can be classified by the relationship between the quantities taken as:

Competing: $\frac{\partial U}{\partial Y} \cdot \frac{\partial X_i}{\partial X_j} < 0$ (2.9)

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[5] Boulding, p. 684.

Independent: $$\frac{\partial U}{\partial Y} \cdot \frac{\partial X_i}{\partial X_j} = 0 \qquad (2.10)$$

Complementary: $$\frac{\partial U}{\partial Y} \cdot \frac{\partial X_i}{\partial X_j} > 0 \qquad (2.11)$$

In this case, item i and item j are classified in relation to what will happen to the quantity consumed of one if the quantity consumed of the other is altered. However, demand theory explicitly specifies that the quantity demanded of a good is a function of the price of that good. Therefore:

$$X_i = f(P_{x_i}) \qquad (2.12)$$

$$X_j = g(P_{x_j}) \qquad (2.13)$$

Assuming that all factors other than the price of X_j are held constant, a change in the purchase rate of X_j is induced by a change in P_{x_j}. If item i and item j are related in some fashion, then the quantity of each item demanded can be altered by a change in consumption of the other. However, the device initiating the change is the price of the substituting item, thus giving the following relation:

$$X_i = f(P_{x_i}, P_{x_j}) \qquad (2.14)$$

$$X_j = g(P_{x_j}, P_{x_i}) \qquad (2.15)$$

The character of the substitution in terms of prices can now be expressed as:

Competing: $$\frac{\partial f}{\partial P_{x_j}}, \frac{\partial g}{\partial P_{x_i}} > 0 \qquad (2.16)$$

Independent: $$\frac{\partial f}{\partial P_{x_j}}, \frac{\partial g}{\partial P_{x_i}} = 0 \qquad (2.17)$$

Complementary: $$\frac{\partial f}{\partial P_{x_j}}, \frac{\partial g}{\partial P_{x_i}} < 0 \qquad (2.18)$$

Up to this point, the role of time in the theoretical foundation and the model specification has been held constant. The durable character of frozen orange concentrate, however, provides for the possible existence of a short-run interaction between a particular price and the length of time this price has been in effect. The cause of this phenomenon could be certain psychological factors peculiar to an individual consumer, or could stem simply from the amount of freezer space available in a customer's home. As in the case of the existence of object expectancies, the underlying cause was not of concern in this study. Rather, the nature of consumer reaction to price over a short time interval was considered to be of interest from the standpoint of obtaining accurate estimates of the demand parameters specified in the model. Thus, there could exist in the marketplace a period following implementation of a price change during which the consumer would not be aware of the new price, or would not fully realize the meaning of the new situation with regard to the product in question or the competing commodities. Previous knowledge concerning the product and its old price would be carried over this time interval and could influence any observation of the effects of the new price. In terms of the consumer behavior theory developed earlier in this chapter, the utility surface of the three categories of frozen orange concentrate would be in a state of flux until the consumer's cognitive processes settled in a new pattern dictated by the relationship between the marginal utilities of various brands and the new price situation.

Concern over this price carryover consideration led to the assumption that manifestation of this phenomenon could occur in any combination of level of purchases and slope of a regression line fit to purchase data, both of which would affect demand elasticity estimates. Provision for the investigation of this effect was made by defining the economic model over four time periods and establishing a procedure for investigating the respective estimating equations over these time intervals. This procedure resulted in the following specification of the complete economic model:

$$Y'_{i1} = \sum_{i=1}^{3} f'_{i1}(X'_{i1}|Z_1) \tag{2.19}$$

$$Y'_{i2} = \sum_{i=1}^{3} f'_{i2}(X'_{i2}|Z_2) \tag{2.20}$$

$$Y'_{i3} = \sum_{i=1}^{3} f'_{i3}(X'_{i3}|Z_3) \tag{2.21}$$

$$Y'_{i4} = \sum_{i=1}^{3} f'_{i4}(X'_{i4}|Z_4) \tag{2.22}$$

where:

Y'_{ij} = quantity of the i^{th} brand sold during the j^{th} time period
$i=1,2,3$
$j=1-4$

f'_{ij} = the relation between the quantity of the i^{th} brand sold during the j^{th} time period and the prices charged for the j^{th} time period
$i=1,2,3$
$j=1-4$

X'_{ij} = the price charged for the i^{th} brand during the j^{th} time period
$i=1,2,3$
$j=1-4$

Z_j, $j=1\text{-}4$ = the effects of exogenous factors during the j^{th} time period assumed to be constant

The main assumption underlying this model was that the quantity of each brand sold was a function of that brand's price and the prices of the other two brands. The exogenous factors--long-run influences on consumer demand related to expectations, consumer attitudes and income --were assumed to remain constant. Short-run disturbances other than those conceptually related to price were not included in the model in that they were controlled by design.

CHAPTER III

FRAMEWORK FOR INTERPRETATION

The development of a procedure for evaluating an economic model is an integral part of a controlled pricing experiment. After specifying the economic model, the next step in this study was to devise a means of estimating the true parameters from data collected in the marketplace. The major complexities in the model structure related to the areas of substitution relationships among the three brand categories and the over-all consideration of price-carryover complications.

Investigation of the response in sales of each brand of concentrate to variations in the price of the two other brands required that a considerable number of price situations be analyzed. It was considered desirable that these situations represent a system of price combinations which encompassed varying levels of all three brand prices. Selection of an analytical framework for data generation and analysis within this structure was a critical part of the design specification.

The question of price-carryover analysis, while supplemental to estimating the demand parameters in the model, was of concern from the standpoint of efficiency of the analytical process. However, the procedures for including this component in the estimating methodology were largely formulated on a judgment basis as to what might occur in the marketplace. Thus, the crystallization of a demand and substitution system within a framework of factors designed to accommodate shifts in this structure over time led to the development of a rather unique experimental procedure.

Components of the Problem

The key word in the description of any economic phenomenon is variation. In order to evaluate the economic model specified in this study, it was necessary to partition this variability into components and measure the average contribution of each component to the over-all fluctuation in the system. However, one of the major problems involved was that all of the disturbance factors could not be accounted for, thereby raising the possibility that the effects of components assumed to be of importance could be overshadowed by a large degree of variability in the data. For this reason, specification of the structure of the statistical model, data generation techniques, and analytical procedures was pursued with a considerable degree of care.

Endogenous factors

In the economic model developed in Chapter II, the quantities purchased of three different brands of frozen orange concentrate were specified as dependent variables. This limitation to only three labels was done for practical reasons concerned with the number of brands of concentrate available to consumers. It was ascertained that three brand classifications based on degree of promotion and firm association utilized by the wholesale and retail trade encompassed all of these brands; thus, the three dependent variables were specified as the respective sales of a single label falling in each of the following categories:

(1) Nationally advertised label--a brand advertised on a national scale and carried by a chain store association, but not affiliated with any chain store group.

(2) Packer label--a brand supplied to a chain store association by an independent distributor, but not advertised by either firm on any scale.

(3) Private label--a brand carried exclusively by an organization of supermarkets, and associated with this organization by local advertising and internal merchandising procedures.

The independent variables in the economic model were specified as the prices for each of the above brand categories. Generation of the values of the dependent variables in association with price increases and decreases across all independent variables was necessary for two reasons. First, in order to estimate the price elasticity of demand for a single concentrate brand, the price of that brand had to vary over a range considerably greater than would normally be experienced in the marketplace. The second reason related to the complete specification of demand elasticities for each brand to include substitution effects. At each price level for a given label, the prices of the other two brands had to vary in decreasing and increasing increments. Therefore, the price variables could not be truly independent, but rather had to be varied according to a carefully developed system.

Another concern in establishing the research procedure was the price-carryover consideration. The specification of where the carryover effect might occur in the estimating model was of primary importance in analyzing the variability attributable to this factor. Possibilities which evolved from deliberations on this point related to potential shifts in the level of consumer purchases as well as changes in the slopes of the price-quantity functions. Conceptually, consumers could respond to a price change by overpurchasing or underpurchasing, eventually settling in a new response pattern free of past price considerations. If this phenomenon of overpurchasing or underpurchasing occurred in the same absolute magnitude at all prices, then the level and not the slope of the demand curve would be affected. However, a change in the slope of the demand curves over time would be indicative of a different price-carryover effect at each new price level.

The final question dealt with in the analytical model was that of store variability. Utilization of retail supermarkets as basic experimental units brought into play such factors as differences in traffic flows through individual stores, changes in behavior between various social and income groups, variations in the physical size of the supermarkets, and differences in merchandising techniques. Control of these factors by design or calculations in the analytical phases was an important factor in making the over-all experimental technique more efficient.

Exogenous factors

In order to provide a framework within which the endogenous variables of the economic model could be examined, a set of controls had to be imposed on certain exogenous influences. One of the major outside forces considered was promotion of the three brands of frozen orange concentrate. This factor included such devices as advertising in the local news media, use of concentrate as a loss leader for other items, special coupon redemption sales, and in-store merchandising techniques utilizing special placards and displays. All of these activities were regarded as an attempt to stimulate customer purchase rates by means not related uniquely to price variation. Use of promotion techniques together with price variations necessary in this study would result in a confounding of the two factor effects, making the estimation of price elasticity coefficients very difficult. Consequently, curtailment of promotion was an important aspect of the data generation phase of the study.

Another exogenous factor, the social and economic environment in which consumers lived in the study area, could not be controlled by design. Changes in consumers' attitudes, preferences and income might possibly be expected to contribute to variations in purchase rates of orange concentrate. Such a development would confound the price effects with the effect of the disturbing exogenous factor. However, the question of variability in socioeconomic factors of this magnitude was assumed to be resolved by the short-run observation period required in a controlled pricing experiment.

The Experimental Design

The procedures used for generating and analyzing data for the demand analysis were conditioned by the model specifications and the principles of the controlled pricing technique. Observations of customer purchases were to be obtained on the retail market under a set of varying price situations. In addition, physical control of the retail merchandising atmosphere with respect to frozen orange concentrate was to be maintained, and an analytical procedure to provide estimates of the parameters in the economic model was needed. Thus, specification of the test price levels, the physical size of the experiment, the necessary controls, and the nature of the statistical analysis of the sales figures all comprised important phases of the complete experimental design.

Treatment selection

In the economic model specified earlier, the quantity of each brand of concentrate was considered to be a function of the three prices. For each quantity, a function of the following order was established:

$$Y_{ij} = \theta(X_{1j},X_{2j},X_{3j}) + e_{ij} \tag{3.1}$$

where:

Y_{ij} = sales of the i^{th} brand at the j^{th} price combination

$\theta(X_{1j},X_{2j},X_{3j})$ = the response surface of Y_{ij} to all values of X_{1j},X_{2j},X_{3j}

X_{1j},X_{2j},X_{3j} = the respective prices of brands 1, 2 and 3 in the j^{th} price combination

e_{ij} = the experimental error of the j^{th} observation

If the exact form of $\theta(X_{1j},X_{2j},X_{3j})$ is not known, this response surface can be estimated by fitting a polynomial to the independent variables.[1] This process consists of defining a region bounded by values of the independent variables that are of interest, and then selecting combinations of these factors for testing according to specifications peculiar to the design used. Several designs have been developed by Box and Hunter[2] for evaluating polynomial surfaces. One of these systems, a central composite rotatable design, was modified for use in this study.

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[1] W. G. Cochran and G. M. Cox, Experimental Design (2nd ed.; New York: John Wiley & Sons, Inc., 1957), p. 335.

[2] Ibid., p. 343.

The most important feature of the basic design selected is its composite character. This particular aspect of the design is an outgrowth of certain problems that evolved when earlier first order designs developed by Box were expanded to include several variables and their second order terms. It was discovered that when second order effects were estimated for "K" factors, the factorial arrangement necessary was of the 3^K series. The generation of data under all possible combinations in this series was found to be unwieldy (for example, $3^4 = 81$ combinations) as the number of factors was increased. Moreover, the same situation developed as the number of levels was expanded beyond three deviations. Fractional replicates and various alternative designs were investigated in an attempt to find a means of estimating second order effects without greatly enlarging the experiment. The end result of this work was the development of the basic composite second order design.

The basis for a composite design encompassing second order effects is a complete two-level factorial experiment. For purposes of continuity in discussion, this arrangement will be specified as having three factors corresponding to the three brand prices in the economic model, while the response term will be considered as the sales of an individual brand. A three-factor design with all factors at two levels is represented in solid lines in Figure 2. Main effects and interactions can be estimated from this design; however, precision in estimating quadratic terms as well as the main effects is gained by adding combinations at the center of the design and on each axis at a constant

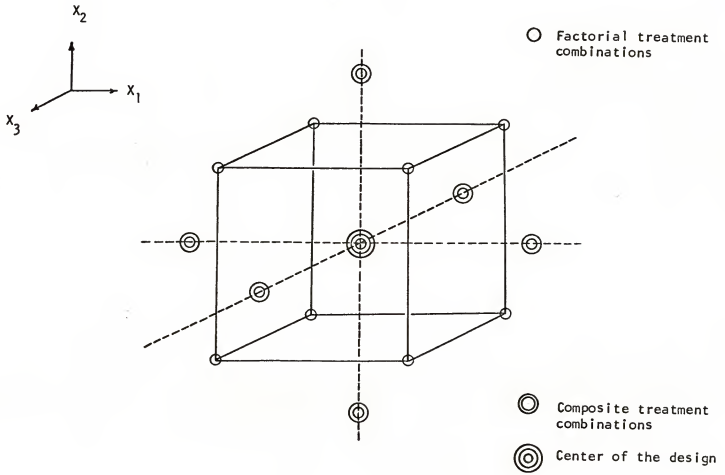
x_2
x_1
x_3
Factorial treatment combinations
Composite treatment combinations
Center of the design

Figure 2.--A central composite design for three factors at two levels distance, α, from the origin.[3] The new treatments shown in dotted lines in Figure 2 give the design a central composite makeup.

In later developments, Tramel, seeking to gain precision in estimation over a wide range of the independent variables in a factorial experiment, oriented the center of a central composite design at what was assumed to be a maximum response for the factors concerned, and then expanded the single solid to three cubes.[4] Tramel also carried the six composite points out to a distance beyond the faces of the largest cube, and retained the treatment combination at the origin. Generation of treatment combinations within such a triple cube arrangement for this project was done in the following manner.

Given that the center of the treatment surface is at coordinates (0,0,0), where x_1, x_2, x_3 are the respective axes, and that the range in either direction of each axis is divided into four equal increments of 1, 2, 3 and 4 units, the equations of four spheres can be written with the center of the region as their center point. These equations are:

$$x_1^2 + x_2^2 + x_3^2 = r_1^2 \tag{3.2}$$

$$x_1^2 + x_2^2 + x_3^2 = r_2^2 \tag{3.3}$$

$$x_1^2 + x_2^2 + x_3^2 = r_3^2 \tag{3.4}$$

$$x_1^2 + x_2^2 + x_3^2 = r_4^2 \tag{3.5}$$

The radii of these spheres are respectively:

$$r_1 = \sqrt{3} = 1.732$$

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[3] Ibid., p. 344.

[4] T. E. Tramel, "A Suggested Procedure for Agronomic-Economic Fertilizer Experiments," Economic and Technical Analysis of Fertilizer Innovations and Resource Use, ed. E. L. Baum, E. O. Heady, J. T. Pesek, and C. G. Hildreth (Ames, Iowa: The Iowa State College Press, 1957).

$$r_2 = \sqrt{12} = 3.464$$
$$r_3 = \sqrt{27} = 5.196$$
$$r_4 = \sqrt{16} = 4.000$$

All points on each solid are now equidistant from the origin of the total surface by the distance specified above. If a cube is inscribed within the first sphere with its surfaces in each respective x_1x_2, x_2x_3 and x_1x_3 plane parallel to that plane, and with sides two units in length, the points on the vertices of this cube provide a set of eight price treatment combinations. If this operation is performed for spheres 1, 2 and 3 with the lengths of the sides of the cubes being 2, 4 and 6 units respectively, the following 24 sets of combinations are obtained from the vertices of the cubes.

Cube 1	Cube 2	Cube 3
+ 1 + 1 - 1	- 2 - 2 - 2	- 3 - 3 - 3
- 1 - 1 + 1	+ 2 - 2 - 2	+ 3 - 3 - 3
+ 1 - 1 + 1	- 2 + 2 - 2	- 3 + 3 - 3
- 1 + 1 + 1	+ 2 + 2 - 2	+ 3 + 3 - 3
+ 1 + 1 + 1	- 2 - 2 + 2	- 3 - 3 + 3
- 1 - 1 - 1	+ 2 - 2 + 2	+ 3 - 3 + 3
+ 1 - 1 - 1	- 2 + 2 + 2	- 3 + 3 + 3
- 1 + 1 - 1	+ 2 + 2 + 2	+ 3 + 3 + 3

These treatment combinations form the basis for a central composite design encompassing three cubes. The size of a complete factorial experiment utilizing three factors at six levels would be 729 combinations. However, in that the treatments on each respective cube are at a constant distance α from the origin, these 24 sets provide an efficient system for measuring main effects and interactions of these three factors over this wide range without testing every possible point on the response surface. In keeping with Tramel's expansion of a basic composite design, the -4, +4 treatments are included only as they appear on each X_i axis, giving a set of treatment combinations at a greater distance from the origin on each axis than other price levels, while actually being on a sphere (3.5) with a smaller radius (3.4). The six resulting combinations are:

-4	0	0
+4	0	0
0	-4	0
0	+4	0
0	0	-4
0	0	+4

By adding the (0,0,0) level to the above values, the complete set of treatments consists of 31 combinations.

Thus, the range of each price variable was designated over nine levels. The total number of combinations for a complete factorial with three factors at nine levels would be too large for practical consideration for any demand study. The main advantages in the use of a triple cube design in selecting price combinations for this study was that efficient estimates of demand parameters could be obtained over this relatively large number of price levels without testing all of the combinations that could possibly be derived.

Price-carryover considerations

The 31 treatment combinations were allocated to a group of retail supermarkets in a restricted manner. The restriction was the desire to expose consumers to a stable price level for some time period following an induced change in prices. No information was available as to what particular course of action should be taken in structuring the experimental design in the stores. Therefore, the problem of selecting a point from which to vary prices for the three brands resolved into the determination of what might constitute a short-run equilibrium of purchase rates among the brands. This equilibrium would not necessarily mean that consumption was fixed at a constant level, but rather that the cognitive processes of selection with reference to price were stable in the consumer's mind. Thus, while proximity to the current market price in the area was desirable, the overriding consideration was that consumers were aware of the new prices being charged, and that the price level position of each brand was essentially the same as that to which they had been accustomed.

Rather than expose the same customers to all price combinations, it was decided that a particular group of customers would be given one price change, allowing time for them to react to the new situation. In conformance with this policy, the group classifications were made in connection with a single store, requiring that a large number of supermarkets be incorporated into the study. Finally, these grocery stores would have to be utilized over a period of weeks that provided sufficient time for investigation of consumer reactions to price changes over time intervals. These intervals, specified on a judgment basis in the economic model were one, two, three and four weeks, respectively.

The basic store design

A randomized complete block design was the basic system for allocating the price treatments to the stores. The model for this design was:

$$Y_{ij} = U + A_i + B_j + E_{ij} \tag{3.6}$$

where:

Y_{ij} = sales response of one brand of frozen orange concentrate associated with the i^{th} treatment combination and the j^{th} block

U = the over-all population mean of the Y_{ij}

A_i = the effect of the i^{th} treatment combination

B_j = the effect of the j^{th} block

E_{ij} = a random disturbance associated with the i^{th} treatment combination and the j^{th} block which is assumed to come from a normally distributed population with mean zero and constant variance σ^2.

Definition of blocks as weeks and single stores within each block as experimental units would have resulted in an experiment requiring 31 stores. Such a design is illustrated in Table 4. Block I in Table 4 was set by randomly assigning the price combinations to the 31 units. Blocks II, III and IV were then set as complete, identical replicates of Block I. Customers shopping in the same store over this 4-week period would be exposed to an initial change from a previous price situation to a test situation which would remain in effect for four consecutive weeks. Detection of significant differences in sales across blocks would then be indicative of the existence of a price-carryover effect within the sales responses.

Expansion of the basic design

The complete block design is accompanied by the assumption

Table 4.--Allocation of triple cube treatment combinations to 31 stores over four weeks for estimating the demand relationships for frozen orange concentrate, 1964

Store #	Block I (Week 1)	Block II (Week 2)	Block III (Week 3)	Block IV (Week 4)
	Treatment combinations			
1	+1 +1 -1	(Repeat arrangement from Block I)		
2	-3 -3 +3			
3	+3 -3 -3			
4	+2 -2 +2			
5	-1 -1 -1			
6	+3 +3 -3			
7	0 +4 0			
8	0 -4 0			
9	-2 +2 +2			
10	+1 -1 +1			
11	+2 -2 -2			
12	0 0 -4			
13	-3 +3 +3			
14	-3 -3 -3			
15	-2 +2 -2			
16	+1 +1 +1			
17	+2 +2 -2			
18	+1 -1 -1			
19	-3 +3 -3			
20	+3 -3 +3			
21	-1 +1 +1			
22	-4 0 0			
23	+4 0 0			
24	+2 +2 +2			
25	-1 +1 -1			
26	-2 -2 -2			
27	-2 -2 +2			
28	0 0 0			
29	+3 +3 +3			
30	-1 -1 +1			
31	0 0 +4			

that within a given block there exists a degree of homogeneity among the experimental units. The efficiency of the design is attributable to the fact that blocking allows removal of the interaction of treatments with experimental units across blocks from the total sums of squares.

In this experiment, two factors prompted modification of the design in Table 4. These were:

(1) Realization that the use of retail stores as experimental units created heterogeneity within blocks.

(2) The desirability of reducing the size of the experiment due to administration and control difficulties.

The major assumption made in the process of modifying the design was that a certain regularity existed with respect to the patterns of customers shopping for frozen orange concentrate. The nature of this assumption was that customers shopping specifically for concentrate were not likely to purchase this commodity more than once during a given week. Moreover, during consecutive weeks, this habit would be maintained causing a recurrence in purchasing at the same relative time within each week.

In view of this assumption, each week was divided into two periods. The breaking point for these periods was between Wednesday and Thursday, and Saturday and Monday. This classification was decided upon because this gave a division of customer traffic and sales volume with regard to biweekly shopping routines as well as equating the number of days in each pricing interval. Thus, the four replications were altered as shown in Table 5 with 32 3-day experimental units per block utilizing 16 stores.

Compensation for differences among the 16 test supermarkets was done by structuring the experimental design and analytical procedure to allow estimation of demand elasticities free of store effects. This was accomplished by adding a base week and two control stores to the basic design. The difference between these added components and the

Table 5.--Revised allocation of triple cube treatment combinations to 16 stores over four weeks for estimating the demand relationships for frozen orange concentrate, 1964

Store #	Period	Week 1	Week 2	Week 3	Week 4
		Treatment combinations			
1	1	+1 +1 -1	(Repeat arrangement from Week 1)		
	2	-3 -3 +3			
2	1	+3 -3 -3			
	2	+2 -2 +2			
3	1	-1 -1 -1			
	2	+3 +3 -3			
4	1	0 +4 0			
	2	0 -4 0			
5	1	-2 +2 +2			
	2	+1 -1 +1			
6	1	+2 -2 -2			
	2	0 0 -4			
7	1	-3 +3 +3			
	2	-3 -3 -3			
8	1	-2 +2 -2			
	2	+1 +1 +1			
9	1	+2 +2 -2			
	2	+1 -1 -1			
10	1	-3 +3 -3			
	2	+3 -3 +3			
11	1	-1 +1 +1			
	2	-4 0 0			
12	1	+4 0 0			
	2	+2 +2 +2			
13	1	-1 +1 -1			
	2	-2 -2 -2			
14	1	-2 -2 +2			
	2	0 0 0			
15	1	+3 +3 +3			
	2	-1 -1 +1			
16	1	0 0 +4			
	2	0 0 0			

regular test stores and weeks is shown in Table 6. The (0,0,0) treatment combination, or mean price level for all brands, was charged in all stores across the base week and across all weeks in the two control stores. In this manner, customers shopping in stores 1-16 were exposed

Table 6.--Expanded allocation of triple cube treatment combinations to 18 stores over one base week and four test weeks for estimating the demand relationships for frozen orange concentrate, 1964

Store #	Period	Base Week	Test Week 1	Test Week 2	Test Week 3	Test Week 4
			Treatment combinations			
1	1	0 0 0	+1 +1 -1			
	2	0 0 0	-3 -3 +3			
2	1	0 0 0	+3 -3 -3			
	2	0 0 0	+2 -2 +2			
3	1	0 0 0	-1 -1 -1			
	2	0 0 0	+3 +3 -3			
4	1	0 0 0	0 +4 0			
	2	0 0 0	0 -4 0			
5	1	0 0 0	-2 +2 +2			
	2	0 0 0	+1 -1 +1			
6	1	0 0 0	+2 -2 -2			
	2	0 0 0	0 0 -4			
7	1	0 0 0	-3 +3 +3			
	2	0 0 0	-3 -3 -3			
8	1	0 0 0	+2 +2 -2			
	2	0 0 0	+1 +1 +1	(Repeat arrangement from Test Week 1)		
9	1	0 0 0	+2 +2 -2			
	2	0 0 0	+1 -1 -1			
10	1	0 0 0	-3 +3 -3			
	2	0 0 0	+3 -3 +3			
11	1	0 0 0	-1 +1 +1			
	2	0 0 0	-4 0 0			
12	1	0 0 0	+4 0 0			
	2	0 0 0	+2 +2 +2			
13	1	0 0 0	-1 +1 -1			
	2	0 0 0	-2 -2 -2			
14	1	0 0 0	-2 -2 +2			
	2	0 0 0	0 0 0			
15	1	0 0 0	+3 +3 +3			
	2	0 0 0	-1 -1 +1			
16	1	0 0 0	0 0 +4			
	2	0 0 0	0 0 0			
17	1	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	2	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
18	1	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0
	2	0 0 0	0 0 0	0 0 0	0 0 0	0 0 0

to the (0,0,0) price level for one week (Table 6). At the beginning of Test Week 1, one of the 31 treatment combinations was charged in each store, and this particular treatment was repeated during period 1 for the remaining four weeks. The same procedure followed for period 2, thus exposing customers shopping in regular patterns in a given store to one price change from the (0,0,0) price level. The complete ramification of this modification is clarified in the analytical procedure.

Analytical procedure

The results of past experiments, coupled with economic theory, precluded the need for a standard analysis of variance for detection of significant treatment effects. The existence of a relationship between the quantity of a product sold and its price was not questioned in this study. Moreover, the reduction in total variation attributable to all of the specified variables was of relatively little importance compared to the contribution of each single factor. The main concern in this study was consideration of structural relationships within the range of the price variables rather than prediction from the total model. The statistical analysis consisted, therefore,of fitting a series of equations using multiple regression and analyzing the individual coefficients within these functions.

Multiple regression

Regression analysis refers to the examination of relationships between variables. In a simple two-variable case in which the direction of influence is known, a model denoting this system can be written as:

$$Y = f(X)$$

where:

Y = the dependent variable

X = the independent variable influencing Y

f = some relation existing between Y and X

If the form of the relationship between Y and X is assumed to be linear, specification of this form in the simplest manner would be:

$$Y = \alpha + BX + u \qquad (3.7)$$

where:

Y = the dependent variable

X = the independent variable

α = an unknown parameter defining the value of Y when $X = 0$

B = an unknown parameter defining the slope of the function as X changes

u = a random variable defined by chance which has a distribution which is normal and independent of the function (3.7) and has mean zero and constant variance σ^2

Estimation of the parameters in (3.7) with more than one pair of (X,Y) values is accomplished by fitting a function to the data subject to the criterion that the sum of the squared deviations of the observed points from the fitted function be a minimum. When this is accomplished, the result is a regression equation, the intercept and slope of which are given by regression coefficients estimating the true parameters. Furthermore, the regression coefficients are in the class of linear unbiased estimators, and having the smallest variance are the best estimators in this class.[5]

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[5] J. Johnston, Econometric Methods (New York: McGraw-Hill Book Co., Inc., 1960), pp. 108-112.

A general linear model involving one dependent variable and "k" independent variables can be specified in compact form by using matrix notation.

$$Y = XB + u \tag{3.8}$$

where:

$$Y = [Y_1 \quad Y_2 \quad \cdots \quad Y_n]'$$

$$X = \begin{bmatrix} 1 & X_{11} & \cdots & X_{1k} \\ 1 & X_{21} & \cdots & X_{2k} \\ \vdots & \vdots & & \vdots \\ 1 & X_{n1} & \cdots & X_{nk} \end{bmatrix}$$

$$B = [B_1 \quad B_2 \quad \cdots \quad B_k]'$$

$$u = [u_1 \quad u_2 \quad \cdots \quad u_n]'$$

and:

Y_n = the value of the dependent variable in the n^{th} observation

X_{nk} = the value of the k^{th} independent variable associated with the n^{th} observation

B_k = the relationship between the dependent variable and the k^{th} independent variable with B_1 being the estimate of the intercept

u_n = the value of the random error component associated with the n^{th} observation

In order to estimate the vector of coefficients B with the method of least squares, the following assumptions must be made.[6]

(1) The values of the independent variables are fixed and known without error.

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[6] *Ibid.*, p. 107.

(2) The matrix X has rank $k < n$.

(3) The following statements characterize the error matrix u.

(a) $E(u) = 0$

(b) $E(uu') = \sigma^2 I_n$

(4) The u_n observations are independent of the X variables.

The main feature of this expansion to k-1 variables is that the analysis of relationships can no longer be accomplished in a single two-dimensional format. The regression equation now measures the effect of many variables on the one dependent factor Y. If the researcher wishes to isolate the relationship between Y and only one X_k, then the regression equation can be differentiated with respect to X_k, and analyzed with all other variables being held constant. This process results in an analysis of the various relationships between the dependent variable and each independent variable in its own dimension. This operation, again recalling the interest only in estimation of the structural aspects of the economic model, provides an objective framework for analyzing the various relationships between the quantities of each brand sold and the respective prices. Moreover, these factor relations are specified in the particular dimension concerned, free, insofar as it is possible to calculate, of the effects of other factors contributing to sales variability.

Specification of the equations

The utilization of multiple regression using the least-squares criterion to estimate the parameters in the economic model in this study was straightforward with one exception. This exception resulted from recognition of heterogeneity of the experimental units. In that no particular pattern could be established for the effects of variability in store size on sales of orange concentrate, it was decided to utilize a difference equation technique of approximation rather than calculation of a regression coefficient for a store variable. The main assumption underlying the use of this method was that while different sales volumes could be expected between stores in which the same price changes were made, the percentage changes in quantity sold would be essentially the same. The removal of store effects from price effects by fitting a difference equation to the data is shown in the following general form.[7]

Define the system of equations:

$$\Delta Y_1 = b_{10} + b_{11}\Delta X_1 + b_{12}\Delta X_2 + b_{13}\Delta X_3 \quad (3.9)$$

$$\Delta Y_2 = b_{20} + b_{21}\Delta X_1 + b_{22}\Delta X_2 + b_{23}\Delta X_3 \quad (3.10)$$

$$\Delta Y_3 = b_{30} + b_{31}\Delta X_1 + b_{32}\Delta X_2 + b_{33}\Delta X_3 \quad (3.11)$$

where:

$\Delta Y_1 = Y^*_{ik} - Y_{ik}$

Y^*_{ik} = the quantity of the i^{th} brand sold in the k^{th} store at the base prices (X^*_i)

Y_{ik} = the quantity of the i^{th} brand sold in the k^{th} store at the design prices (X_i)

b_i = regression constants

$\Delta X_i = X^*_i - X_i$

X^*_i = the base prices

X_i = the design prices

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[7] This difference equation technique is based on thoughts provided by V. G. Hurt, Department of Agricultural Economics, Mississippi State University, State College, Mississippi.

The estimating equations can now be written for brand 1 in the following form:

$$Y^*_{1k} - Y_{1k} = b_{10} + b_{11}(X^*_i - X_1) + b_{12}(X^*_2 - X_2) + b_{13}(X^*_3 - X_3) \qquad (3.12)$$

or:

$$Y_{1k} = Y^*_{1k} - b_{10} - b_{11}X^*_1 + b_{11}X_1 - b_{12}X^*_2 + b_{12}X_2 - b_{13}X^*_3 + b_{13}X_3 \qquad (3.13)$$

$$Y_{1k} = [Y^*_{1k} - b_{10} - b_{11}X^*_1 - b_{12}X^*_2 - b_{13}X^*_3] + b_{11}X_1 + b_{12}X_2 + b_{13}X_3 \qquad (3.14)$$

If the X^*_1, X^*_2, X^*_3 are held constant as base prices in all stores during the base period, then Y^*_{1k} functions as an adjustment in the intercept for store differences.

While the difference equation approach applied to data generated within the design shown in stores 1-16 in Table 6 allows for estimation of price effects across weeks, there is an inherent danger that some short-run regional exogenous influence could function as a disturbing factor across all stores. If this situation were to occur in one particular week, a shift in sales across all stores could be incorrectly attributed to a time-price interaction and give an erroneous indication of a price-carryover effect for that week. Addition of the base week and control stores to the basic store design provided the system in which this possibility could be compensated for in the statistical analysis.[8] This procedure involved the adjustment of sales information observed in the 16 test stores through the use of an index calculated from these added components. These tabular adjustments were made on sales of each brand in the same manner as shown for one label in Table 7. The price level of this brand remained unchanged in the control store. Therefore, the actual sales during the ensuing periods within weeks were assumed to remain relatively constant. Any deviation in actual sales for the control store would be attributed to exogenous influences which were assumed to be operating across all stores. Thus, the expected index calculated for the control store multiplied by the base week sales in the test stores would result in the amount expected to be sold in the test stores during the test weeks if the prices in the test stores had been unchanged. This calculation prior to fitting the regression equations was assumed to be efficient enough in compensating for exogenous influences across weeks in all stores so that a variable for this factor was not necessary in the analysis. With this final

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[8] Ibid., Hurt.

Table 7.--Base week and control store adjustment used in estimating the demand relationships for frozen orange concentrate, 1964

Store	Sales in Base Week, Period 1	Test Week 1, Period 1 Sales		Test Week 2, Period 1 Sales	
		Actual	Expected	Actual	Expected
Control	100	105	(1.05)	95	(.95)
Test 1	90	90	94.5	85	85.5
Test 2	110	150	115.5[a]	145	104.5[b]
. . .					

[a] (110)(1.05) = 115.5

[b] (110)(.95) = 104.5

consideration in mind, the assumptions underlying the over-all expanded experimental design were stated as:

(1) Store differences could be removed by adjustment; i.e., treatment effects in terms of actual or percentage variations from base sales or expected sales could be the same for all stores having the same price variation.

(2) Traffic differences between stores could be removed by expressing all quantities sold in terms of a purchase rate for each experimental unit.

(3) The effects of exogenous forces could be removed from the sales data by calculating quantities sold in terms of an index multiple computed from base sales and deviations recorded in the control stores.

(4) There would not be any confounding of treatment effects with period effects across weeks. Customers were assumed to maintain their ordinary shopping habits for the duration of the study.

Analytical Model I.--The first equation fit to the sales data utilized first differences in natural logarithms for the dependent variable and the linear effects of the independent variables. In general terms, the difference in actual and expected sales was related to the difference between the base price and test price for a given store, with all figures transformed to natural logarithms. In matrix notation, this first equation format was:

$$Y_1 = X_1\hat{B}_1 \tag{3.15}$$

where:

$$Y_1 = [Y_{i-1} \quad Y_{i-2} \quad \cdots \quad Y_{i-31}]'$$

$$X_1 = \begin{bmatrix} 1 & \ln X_{1-1} & \ln X_{2-1} & \ln X_{3-1} \\ 1 & \ln X_{1-2} & \ln X_{2-2} & \ln X_{3-2} \\ \cdot & \cdot & & \\ \cdot & \cdot & & \\ \cdot & \cdot & & \\ 1 & \ln X_{1-31} & \ln X_{2-31} & \ln X_{3-31} \end{bmatrix}$$

$$\hat{B}_1 = [\ln b_{10} \quad b_{11} \quad b_{12} \quad b_{13}]'$$

Fitting (3.15) to the sales data for brands 1, 2 and 3 and across weeks 2, 3, 4 and 5 would result in the following equations:

$$Y_{1-kn} = b_{10-k} + b_{11-k}X_{1-kn} + b_{12-k}X_{2-kn} + b_{13-k}X_{3-kn} \quad (3.16)$$

$$Y_{2-kn} = b_{20-k} + b_{21-k}X_{1-kn} + b_{22-k}X_{2-kn} + b_{23-k}X_{3-kn} \quad (3.17)$$

$$Y_{3-kn} = b_{30-k} + b_{31-k}X_{1-kn} + b_{32-k}X_{2-kn} + b_{33-k}X_{3-kn} \quad (3.18)$$

where:

k = a subscript denoting the k^{th} week which takes on the values 2, 3, 4 and 5

Y_{i-kn} = [ln (expected quantity sold of the i^{th} brand during the k^{th} test week in the store having the n^{th} price combination) - ln (actual quantity sold of the i^{th} brand during the k^{th} week in the store having the n^{th} price combination)]

b_{i0-k} = estimate of the intercept for the i^{th} brand during the k^{th} test week

b_{ij-k} $(i=j)$ = estimate of the coefficient of price elasticity of demand for the i^{th} brand during the k^{th} test week

b_{ij-k} $(i \neq j)$ = estimate of the coefficient of cross-price elasticity of demand between the i^{th} brand's sales and the price of the j^{th} brand during the k^{th} week

X_{i-kn} = [ln (base price for the i^{th} brand) - ln (actual price charged for the i^{th} brand during the k^{th} week in the store having the n^{th} price combination)]

As a part of the price-carryover analysis, in addition to the equation sets calculated for individual weeks, a similar three equation system for all brands was calculated for all weeks as a group. Transformation of the dependent and independent variables to natural logarithms in this model was primarily to facilitate interpretation of the elasticity coefficients.

Analytical Model II.--The equations fit in the second model resulted from the expansion of the difference equation discussion mentioned previously to the extent of investigating the square and cross-product terms as allowed in the central composite design. Enlarging (3.12) to include square and cross-product terms gave:

$$\begin{aligned} Y^*_{1k} - Y_{1k} = {} & b_{10} + b_{11}(X^*_1 - X_1) + b_{12}(X^*_2 - X_2) + b_{13}(X^*_3 - X_3) \\ & + b_{14}(X^*_1 - X_1)^2 + b_{15}(X^*_2 - X_2)^2 + b_{16}(X^*_3 - X_3)^2 \\ & + b_{17}[(X^*_1 - X_1)(X^*_2 - X_2)] + b_{18}[(X^*_1 - X_1)(X^*_3 - X_3)] \\ & + b_{19}[(X^*_2 - X_2)(X^*_3 - X_3)] \end{aligned} \tag{3.19}$$

where:

Y^*_{1k} = the quantity of brand 1 sold in the k^{th} store at the base prices (X^*_i)

Y_{1k} = the quantity of brand 1 sold in the k^{th} store at the test prices (X_i)

b_{ij} = regression coefficients

X^*_i = the base prices

Expanding (3.19) and collecting terms:

$$\begin{aligned} Y^*_{1k} - Y_{1k} = {} & b_{10} + b_{11}X^*_1 + b_{12}X^*_2 + b_{13}X^*_3 + b_{14}X^{*2}_1 \\ & + b_{15}X^{*2}_2 + b_{16}X^{*2}_3 + b_{17}X^*_1X^*_2 + b_{18}X^*_1X^*_3 \\ & + b_{19}X^*_2X^*_3 + (-b_{11} - 2bX^*_1 - b_{17}X^*_2 - b_{18}X^*_3)X_1 \\ & + (-b_{12} - 2b_{15}X^*_2 - b_{17}X^*_1 - b_{19}X^*_3)X_2 \\ & + (-b_{13} - 2b_{16}X^*_3 - b_{18}X^*_1 - b_{19}X^*_2)X_3 + b_{14}X_1^2 \\ & + b_{15}X_2^2 + b_{16}X_3^2 + b_{17}X_1X_2 + b_{18}X_1X_3 \\ & + b_{19}X_2X_3 \end{aligned} \tag{3.20}$$

Subtracting Y^*_{1k} from both sides of the equation (3.20) and multiplying both sides by -1 resulted in the following form:

$$\begin{aligned} Y_{1k} &= [Y^*_{1k} - b_{10} - b_{11}X^*_1 - b_{12}X^*_2 - b_{13}X^*_3 - b_{14}X^{*2}_1 \\ &\quad - b_{15}X^{*2}_2 - b_{16}X^{*2}_3 - b_{17}X^*_1X^*_2 - b_{18}X^*_1X^*_3 \\ &\quad - b_{19}X^*_2X^*_3] + b'_1X_1 + b'_2X_2 + b'_3X_3 - b_{14}X_1^2 \\ &\quad - b_{15}X_2^2 - b_{16}X_3^2 - b_{17}X_1X_2 - b_{18}X_1X_3 \\ &\quad - b_{19}X_2X_3 \end{aligned} \qquad (3.21)$$

where:

$$\begin{aligned} b'_{11} &= b_{11} + 2b_{14}X^*_1 + b_{17}X^*_2 + b_{18}X^*_3 \\ b'_{12} &= b_{12} + 2b_{15}X^*_2 + b_{17}X^*_1 + b_{19}X^*_3 \\ b'_{13} &= b_{13} + 2b_{16}X^*_3 + b_{18}X^*_1 + b_{19}X^*_2 \end{aligned}$$

thus allowing estimation of the price coefficients b_{11}, b_{12}, b_{13} by subtraction of the respective base components from b'_1, b'_2 and b'_3. The intercept estimate incorporates the base week sales values in with the remaining base price components in brackets in equation (3.21); therefore, resulting in the dependent variable merely being sales during the test weeks.

The equation form fit under this new procedure was:

$$Y_{11} = X_{11}\hat{B}_{11} \qquad (3.22)$$

where:

$$Y_{11} = [Y_{i-1} \quad Y_{i-2} \quad \cdots \quad Y_{i-31}]'$$

$$X_{11} = \begin{bmatrix} 1 & X_{i-1} - X^*_i & .2396 & \cdots \\ 1 & \cdot & & \\ \cdot & \cdot & & \\ \cdot & \cdot & & \\ \cdot & & & \\ 1 & \text{(Tramel's modified triple cube X matrix)}^{9} & & \end{bmatrix}$$

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[9] Tramel, p. 172.

$$\hat{B}_{11} = [b'_{10} \quad b'_{11} \quad \cdots \quad b_{19}]'$$

Fitting (3.22) to the sales data for brands 1, 2 and 3, and across weeks 2, 3, 4 and 5, results in the following equations:

$$\begin{aligned} Y_{1-kn} = {} & b'_{10-k} + b'_{11-k}X_{1-kn} + b'_{12-k}X_{2-kn} + b'_{13-k}X_{3-kn} \\ & + b_{14-k}X_{1-kn}^{2} + b_{15-k}X_{2-kn}^{2} + b_{16-k}X_{3-kn}^{2} \\ & + b_{17-k}X_{1-kn}X_{2-kn} + b_{18-k}X_{1-kn}X_{3-kn} \\ & + b_{19-k}X_{2-kn}X_{3-kn} \end{aligned} \tag{3.23}$$

$$\begin{aligned} Y_{2-kn} = {} & b'_{20-k} + b'_{21-k}X_{1-kn} + b'_{22-k}X_{2-kn} + b'_{23-k}X_{3-kn} \\ & + b_{24-k}X_{1-kn}^{2} + b_{25-k}X_{2-kn}^{2} + b_{26-k}X_{3-kn}^{2} \\ & + b_{27-k}X_{1-kn}X_{2-kn} + b_{28-k}X_{1-kn}X_{3-kn} \\ & + b_{29-k}X_{2-kn}X_{3-kn} \end{aligned} \tag{3.24}$$

$$\begin{aligned} Y_{3-kn} = {} & b'_{30-k} + b'_{31-k}X_{1-kn} + b'_{32-k}X_{2-kn} + b'_{33-k}X_{3-kn} \\ & + b_{34-k}X_{1-kn}^{2} + b_{35-k}X_{2-kn}^{2} + b_{36-k}X_{3-kn}^{2} \\ & + b_{37-k}X_{1-kn}X_{2-kn} + b_{38-k}X_{1-kn}X_{3-kn} \\ & + b_{39-k}X_{2-kn}X_{3-kn} \end{aligned} \tag{3.25}$$

where:

k = a subscript denoting the kth week which takes on the values 2, 3, 4 and 5

Y_{i-kn} = the quantity sold adjusted by a control store index of the ith brand during the kth week in the store having the nth price combination

b'_{i0-k} = summary component estimate of the quantity intercept for the ith brand during the kth week

b'_{ij-k} $(i=j)$ = summary component estimate of the regression coefficient relating the quantity of the ith brand sold during the kth week to the price of the ith brand

b'_{ij-k} $(i \neq j)$ = summary component estimate of the regression coefficient relating the quantity of the ith brand sold during the kth week to the price of the jth brand

b_{ij-k} = estimate of the regression coefficient relating the sales of the i^{th} brand during the k^{th} week to the square and cross-product term of the prices of the i^{th} and j^{th} brands

X_{i-kn} = the price of the i^{th} brand charged during the k^{th} week in the n^{th} treatment combination

This procedure will also incorporate a price-carryover analysis involving calculation of a set of equations using the above format and data summed over all weeks.

Analytical Model III.--The third equation system fit to the data was the same as analytical Model I with two exceptions. First, the data was not transformed to natural logarithms, and second, the square and cross-product terms of the independent variables were added to the equations. The reason for not affecting a logarithmic transformation was concerned with the methodology of logarithmic versus arithmetic functions in approximating demand relationships. In this study, this concern was purely from the standpoint of a comparison of estimates of the same regression coefficients, and the possible meanings to be derived from these alternative approximations. The addition of square and cross-product terms to the polynomial representing the response surface for a single dependent variable was in line with the full utilization of the triple cube design as discussed earlier in this chapter for composite designs.

CHAPTER IV

CONDUCT OF THE EXPERIMENT

Development of an economic model and analytical procedures for evaluating the model comprises the initial step in a controlled pricing experiment. The next phase is the generation of data through observation within a controlled market system in which the experimental design requirements are implemented. The physical conduct of this study took place on the retail market within a framework of operations which provided the type of observational data necessary for analysis. The considerations involved in this implementation related to translation of the experimental design into a physical data generation procedure. This operation involved test site and store selection, transference of the treatment combinations to actual price levels, allocation of the prices to the retail supermarkets, and recording customer purchase rates under these prices in accordance with certain basic control policies peculiar to the experiment.

The Test Site

Regional description

Selection of the region in which this study was conducted was mainly a process of locating an area in which the required number of experimental units could be obtained and controlled. It was judged desirable that the 18 test stores, necessary for satisfaction of the design requirements, also covered a range of population which would enhance the representativeness of the sales reactions to be obtained. Several areas in the eastern continental United States were investigated, with contacts being made with various chain store operations. After considerable negotiation, an organization with stores located in the Dayton-Cincinnati area of Ohio extended the degree of cooperation necessary to carry on the data generation process. Thus, the boundaries characterized by the 18 test stores encompassed an urban-rural complex with a population numbering over two million people.[1]

The test stores

The distribution of the supermarkets used in urban areas within the test region was as follows:

Dayton	11 stores
Fairborn	1 store
Miamisburg	1 store
Middletown	1 store
Hamilton	2 stores
Cincinnati	2 stores

The Dayton area was the prime study region having the largest number of stores. These units, located within Dayton and its suburbs, provided a good cross-section of the population by income areas. In all cases, the stores were large and modern in design with 12 of them located in shopping centers.

The average amount of customer traffic through these supermarkets during the 5-week period of study controls was tabulated at

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[1] U. S. Bureau of the Census, Statistical Abstract of the United States: 1963.

38,500 customers per store (Table 8). The range of this traffic average was from a low of 2,645 customers during period 1 to a high of 5,350 persons for period 2. In terms of the 3-day test periods, 36 percent of the total traffic count was recorded during period 1, with 64 percent of the customers counted in period 2 over all stores. Average dollar sales over the 18 test stores was computed as $208,269.98 per store for the duration of the study, with 23 percent of the volume registered during period 1 and 77 percent accounted for in period 2 (Table 9).

Table 8.--Average customer traffic distribution per store by week and period recorded during the data generation phase for estimating the demand relationships for frozen orange concentrate, 1964

Week	Period average		Weekly average
	1	2	
1	2,645	4,859	7,504
2	2,653	4,517	7,170
3	3,044	4,752	7,796
4	2,790	5,033	7,823
5	2,857	5,350	8,207
Total average	13,989	24,511	38,500

Field Operations

The actual conduct of this study was carried out within an atmosphere of controls which was necessary from the standpoint of collecting accurate information for demand analysis. This was not

Table 9.--Average dollar sales distribution per store by week and period recorded during the data generation phase for estimating the demand relationships for frozen orange concentrate, 1964

Week	Period average		Weekly average
	1	2	
1	8,616.28	34,255.07	42,871.35
2	8,588.27	29,480.17	38,068.44
3	10,950.59	32,425.29	43,375.88
4	9,099.67	31,813.80	40,913.47
5	10,047.36	32,993.48	43,040.84
Total average	47,302.17	160,997.81	208,269.98

accomplished in such a manner as to call attention to the existence of controls on the retail market, but rather was pursued with the objective of causing as little interruption as possible in the normal routine in the supermarkets involved. Thus, application of the experimental conditions regarding pricing, merchandising and frozen orange concentrate inventory procedures in the test stores was a matter of merging the existing market atmosphere and these requirements into one operation.

Price selection

The data generation period in the 18 supermarkets used was during the months of May and June, 1964. Prior to the initiation of the experiment, the market for frozen orange concentrate in the area was characterized by relatively high prices for all brands as a result of the severe freeze incurred in Florida during the 1962-63 season.

The cooperating agency carried a wide variety of concentrate brands in varying amounts between stores. There were three distinct brands, however, which were within the study classifications and common to all stores. Moreover, a standard set of pricing and merchandising techniques was applied to these three brand classifications pertaining to a nationally advertised label, an independent packer label and a private label.

Specification of price treatments to be applied in the test stores in conformance with the experimental design was done within the framework of standard retail pricing policy and the design requirements. It was ascertained that the national label characteristically was the highest in price, while the packer and private brands were sold at close to the same differential below the national brand's price level. In line with this policy, retail prices charged by the cooperating agency per 6-ounce can of concentrate prior to the study were:

(1) National brand	35 cents
(2) Packer brand	28 cents
(3) Private brand	29 cents--Dayton area 30 cents--area south of Dayton

While these prices could be regarded as market prices from which the test treatments would normally deviate, the prospect of establishing these values as the average or base price level for the three study brands was not considered feasible. A fairly wide range of prices which would be considered relevant from the standpoint of future decision-making in the orange industry was needed, with the base prices for each brand necessarily being at the center of this range. The prevailing market prices, in that they reflected a carryover of policies resultant from an abnormal freeze year, were regarded as too high to be used as average prices. Rather, the position of these prices seemed more readily acceptable in the upper extremes of the test price ranges.

With respect to the magnitude of the test price range and the number of levels within this range for each brand, the triple cube design specifications given earlier established four deviations above and below the average price values. These levels were regarded as prices per 6-ounce can, and when set at 1-, 2- and 3-cent differentials, resulted in a range of 8 cents, 16 cents and 24 cents from low to high prices per can respectively. The 16-cent range or 2-cent differential was selected as providing a portion of the demand curve large enough to allow successful evaluation of the economic model. Moreover, in keeping with the standard pricing differential policy used in the retail supermarkets and the desire that the test price range for each brand approximate a market situation generally lower in price level than the prevailing condition, this 16-cent range was established for the test brands within the standard retail price spread among brands (Table 10).

The values listed in Table 10 for the three brands provided the basis for specification of the exact 31 treatment combinations utilized in the over-all store design. The derivation of these combinations from the triple cube complex is shown in Table 11. For example, the (0,0,0) treatment combination corresponds to the base or average price values of 29 cents, 25 cents and 25 cents respectively for the national, packer and private brands, while one of the corner treatments on the first cube, (+1,-1,+1) corresponds to the prices 31 cents, 23 cents and 27 cents in the same brand order.

Table 10.--Price ranges, treatment levels, and brand positions used in estimating the demand relationships for frozen orange concentrate, 1964

Nationally advertised label			Packer label			Private label		
Treatment level	Deviation	Price	Treatment level	Deviation	Price	Treatment level	Deviation	Price
	Cents			Cents			Cents	
+4	+8	37						
+3	+6	35[a]						
+2	+4	33	+4	+8	33	+4	+8	33
+1	+2	31	+3	+6	31	+3	+6	31[a]
0	0	29[b]	+2	+4	29[a]	+2	+4	29
-1	-2	27	+1	+2	27	+1	+2	27
-2	-4	25	0	0	25[b]	0	0	25[b]
-3	-6	23	-1	-2	23	-1	-2	23
-4	-8	21	-2	-4	21	-2	-4	21
			-3	-6	19	-3	-6	19
			-4	-8	17	-4	-8	17

[a] The underlined deviations indicate the region of the market price for each brand before the study began.

[b] The 0 deviations denote the mid-range, or mean prices for the price spread used for each brand.

The series of 31 treatments was by no means a total listing of the number of factorial combinations that could be derived from the price levels given in Table 10. These treatments were regarded as the most practical and efficient comparisons which could be selected for approximating the surface bounded by the extreme price levels, while the 2-cent pricing differential was assumed to provide a satisfactory portion of the demand functions for analysis.

Allocation of prices to the test stores

The basic design utilizing 16 stores and two periods within each week provided 32 units per block to receive the test price

Table 11.--Price comparisons used in estimating the demand relationships for frozen orange concentrate, 1964

Combination	Nationally advertised label		Packer label		Private label	
	Design level and corresponding prices in cents per 6-ounce can					
	Level indication	Price	Level indication	Price	Level indication	Price
1	0	29	-4	17	0	25
2	-3	23	-3	19	-3	19
3	-3	23	-3	19	+3	31
4	+3	35	-3	19	-3	19
5	+3	35	-3	19	+3	31
6	-2	25	-2	21	-2	21
7	-2	25	-2	21	+2	29
8	+2	33	-2	21	-2	21
9	+2	33	-2	21	+2	29
10	-1	27	-1	23	-1	23
11	-1	27	-1	23	+1	27
12	+1	31	-1	23	-1	23
13	+1	31	-1	23	+1	27
14	-4	21	0	25	0	25
15	0	29	0	25	-4	17
16	0	29	0	25	0	25
17	0	29	0	25	+4	33
18	+4	37	0	25	0	25
19	-1	27	+1	27	+1	27
20	+1	31	+1	27	+1	27
21	-1	27	+1	27	-1	23
22	+1	31	+1	27	-1	23
23	-2	25	+2	29	-2	21
24	-2	25	+2	29	+2	29
25	+2	33	+2	29	+2	29
26	+2	33	+2	29	-2	21
27	-3	23	+3	31	+3	31
28	-3	23	+3	31	-3	19
29	+3	35	+3	31	-3	19
30	+3	35	+3	31	+3	31
31	0	29	+4	33	0	25

combinations. In that only 31 treatments were specified, the mean price level was repeated once in order to fill the remaining period. The maximum amount of variation for one brand's price in a given store was set at 4 cents or two treatment levels. The reason for this stipulation was the realization that considerably large price changes for no apparent reason could possibly jeopardize the cooperating agency's customer relations. In that no loss of information was envisioned by adopting this restriction, 16 pairs of treatment combinations were established within this 4-cent differential requirement for each brand and randomly allocated to the 16 test stores in the first test week. The result of this procedure for test week 1 was replicated in test weeks 2, 3 and 4, giving the basic store design shown in Table 12.

The two control stores necessary for accommodating store differences and short-run nonprice variation in concentrate sales in the general analytical procedure were selected on a judgment basis in consultation with the executives of the cooperating agency. The supermarkets chosen were regarded as adequate representatives of the 16 stores which received the 31 test price combinations. In conjunction with the role assigned these two stores in the experimental design, the base prices were applied to the three brands of concentrate being sold in these supermarkets for the duration of the study (Table 13).

The difference equation analytical procedure required that only one week be used as a base week during which the mean price level would be charged across all stores for all brands. The critical nature of customer awareness of the mean price level and the study pricing system led to the addition of a second base week to the total store design. The shift from prevailing retail market prices to the mean price combination for each brand was presented to customers shopping in the 18 supermarkets for a 2-week period prior to implementation of the varying test price situations. This expansion of

Table 12.--Basic design for allocation of prices per 6-ounce can to three brands of frozen orange concentrate used in estimating the demand relationships for frozen orange concentrate, 1964

Store #	Period	Test Week 1	Test Week 2	Test Week 3	Test Week 4
		A B C*	A B C	A B C	A B C
1	1	29-25-25	29-25-25	29-25-25	29-25-25
	2	27-27-23	27-27-23	27-27-23	27-27-23
2	1	23-31-31	23-31-31	23-31-31	23-31-31
	2	25-29-29	25-29-29	25-29-29	25-29-29
3	1	29-25-25	29-25-25	29-25-25	29-25-25
	2	27-27-27	27-27-27	27-27-27	27-27-27
4	1	29-25-17	29-25-17	29-25-17	29-25-17
	2	33-21-21	33-21-21	33-21-21	33-21-21
5	1	33-29-21	33-29-21	33-29-21	33-29-21
	2	29-33-25	29-33-25	29-33-25	29-33-25
6	1	31-23-27	31-23-27	31-23-27	31-23-27
	2	35-19-31	35-19-31	35-19-31	35-19-31
7	1	23-19-19	23-19-19	23-19-19	23-19-19
	2	27-23-23	27-23-23	27-23-23	27-23-23
8	1	35-31-19	35-31-19	35-31-19	35-31-19
	2	31-27-23	31-27-23	31-27-23	31-27-23
9	1	35-19-19	35-19-19	35-19-19	35-19-19
	2	31-23-23	31-23-23	31-23-23	31-23-23
10	1	29-17-25	29-17-25	29-17-25	29-17-25
	2	25-21-21	25-21-21	25-21-21	25-21-21
11	1	37-25-25	37-25-25	37-25-25	37-25-25
	2	33-29-29	33-29-29	33-29-29	33-29-29
12	1	29-25-33	29-25-33	29-25-33	29-25-33
	2	33-21-29	33-21-29	33-21-29	33-21-29
13	1	27-23-27	27-23-27	27-23-27	27-23-27
	2	23-19-31	23-19-31	23-19-31	23-19-31
14	1	31-27-27	31-27-27	31-27-27	31-27-27
	2	35-31-31	35-31-31	35-31-31	35-31-31
15	1	21-25-25	21-25-25	21-25-25	21-25-25
	2	25-21-29	25-21-29	25-21-29	25-21-29
16	1	25-29-21	25-29-21	25-29-21	25-29-21
	2	23-31-19	23-31-19	23-31-19	23-31-19

*Brand A--nationally advertised; brand B--packer; brand C--private.

the design did not alter the analytical procedure in that sales figures from the base week immediately preceding the first test week were still to be the only data used in the prescribed analysis.

Table 13.--Total store design for allocation of prices per 6-ounce can to three brands of frozen orange concentrate used in estimating the demand relationships for frozen orange concentrate, 1964

Store #	Period	Base Week 1	Base Week 2	Test Week 1	Test Week 2	Test Week 3	Test Week 4
		A B C*	A B C	A B C	A B C	A B C	A B C
1	1	29-25-25	29-25-25	29-25-25	29-25-25	29-25-25	29-25-25
	2	29-25-25	29-25-25	27-27-23	27-27-23	27-27-23	27-27-23
2	1	29-25-25	29-25-25	23-31-31	23-31-31	23-31-31	23-31-31
	2	29-25-25	29-25-25	25-29-29	25-29-29	25-29-29	25-29-29
3	1	29-25-25	29-25-25	29-25-25	29-25-25	29-25-25	29-25-25
	2	29-25-25	29-25-25	27-27-27	27-27-27	27-27-27	27-27-27
4	1	29-25-25	29-25-25	29-25-17	29-25-17	29-25-17	29-25-17
	2	29-25-25	29-25-25	33-21-21	33-21-21	33-21-21	33-21-21
5	1	29-25-25	29-25-25	33-29-21	33-29-21	33-29-21	33-29-21
	2	29-25-25	29-25-25	29-33-25	29-33-25	29-33-25	29-33-25
6	1	29-25-25	29-25-25	31-23-27	31-23-27	31-23-27	31-23-27
	2	29-25-25	29-25-25	35-19-31	35-19-31	35-19-31	35-19-31
7	1	29-25-25	29-25-25	23-19-19	23-19-19	23-19-19	23-19-19
	2	29-25-25	29-25-25	27-23-23	27-23-23	27-23-23	27-23-23
8	1	29-25-25	29-25-25	35-31-19	35-31-19	35-31-19	35-31-19
	2	29-25-25	29-25-25	31-27-23	31-27-23	31-27-23	31-27-23
9	1	29-25-25	29-25-25	35-19-19	35-19-19	35-19-19	35-19-19
	2	29-25-25	29-25-25	31-23-23	31-23-23	31-23-23	31-23-23
10	1	29-25-25	29-25-25	29-17-25	29-17-25	29-17-25	29-17-25
	2	29-25-25	29-25-25	25-21-21	25-21-21	25-21-21	25-21-21
11	1	29-25-25	29-25-25	37-25-25	37-25-25	37-25-25	37-25-25
	2	29-25-25	29-25-25	33-29-29	33-29-29	33-29-29	33-29-29
12	1	29-25-25	29-25-25	29-25-33	29-25-33	29-25-33	29-25-33
	2	29-25-25	29-25-25	33-21-29	33-21-29	33-21-29	33-21-29
13	1	29-25-25	29-25-25	27-23-27	27-23-27	27-23-27	27-23-27
	2	29-25-25	29-25-25	23-19-31	23-19-31	23-19-31	23-19-31

14	1	29-25-25	29-25-25	31-27-27	31-27-27	31-27-27	31-27-27
	2	29-25-25	29-25-25	35-31-31	35-31-31	35-31-31	35-31-31
15	1	29-25-25	29-25-25	21-25-25	21-25-25	21-25-25	21-25-25
	2	29-25-25	29-25-25	25-21-29	25-21-29	25-21-29	25-21-29
16	1	29-25-25	29-25-25	25-29-21	25-29-21	25-29-21	25-29-21
	2	29-25-25	29-25-25	23-31-19	23-31-19	23-31-19	23-31-19
17**	1	29-25-25	29-25-25	29-25-25	29-25-25	29-25-25	29-25-25
	2	29-25-25	29-25-25	29-25-25	29-25-25	29-25-25	29-25-25
18**	1	29-25-25	29-25-25	29-25-25	29-25-25	29-25-25	29-25-25
	2	29-25-25	29-25-25	29-25-25	29-25-25	29-25-25	29-25-25

*Brand A--nationally advertised; brand B--packer; brand C--private.

**Control store.

The total store design, utilizing 16 test stores and two control stores over a period of six weeks, is illustrated in Table 13. In essence, the experimental procedure with regard to pricing was to initially charge the base price treatment (29 cents-25 cents-25 cents) over periods 1 and 2 and all stores for two base weeks. Beginning with period 1, test week 1, 15 of the test price combinations plus the base price combination were charged in the 16 test stores, with these same prices being repeated during period 1 in the respective supermarkets for the following test weeks. The remaining 16 price treatments were charged during period 2 in the 16 test stores, and subsequently replicated in these stores for test weeks 2, 3 and 4, while the two control stores carried the base price combinations for all base and test weeks.

Store operations

Daily sales of each brand of frozen orange concentrate in the 18 supermarkets were recorded by enumerators hired and trained specifically for this study. Each enumerator was responsible for implementing the physical aspects of the data generation procedure in two stores. The major aspects of these procedures were:

(1) Management of the displays and storage facilities for the three brands of frozen orange concentrate in each store.

(2) Pricing the various brands of orange concentrate at the levels specified by the experimental design for each store and time period (Table 13).

(3) The recording of inventory and sales figures for the test brands of concentrate on a daily basis in each store.

Display management.--The three brands chosen as representative of the nationally advertised, packer and private labels were the only frozen orange concentrates offered for sale in the 18 supermarkets.

Each store carried all three brands in the following can sizes:

(1) Nationally advertised	6-ounce can 12-ounce can
(2) Packer	6-ounce can 12-ounce can
(3) Private	6-ounce can 6-ounce can in a 6-pack carton 12-ounce can

Investigation of display procedures in all prospective test stores before the study resulted in the specification of a minimum display to be established in all supermarkets. This display, shown in Figure 3, was expanded in some of the supermarkets which normally had large offerings of concentrate for customers, but in no case was the number of rows per brand reduced. In addition, the left-to-right ordering of brands as customers faced the display was identical for all stores, with individual cans and multipacks arranged in row stacks. Uniformity in pricing was also required for the three brands in all of the test stores. The design prices shown in Table 10 were expanded to multiunit prices which were in keeping with normal retail pricing procedures. This system resulted in 6-ounce cans being priced in units of two, 12-ounce cans in single units, and the 6-can multipack as a single unit. The prices charged were all multiples of a single 6-ounce can price lowered by one-half cent (Table 14). These prices were stamped on placards which bore printed indication of the respective label and can size. The placards were then mounted over the corresponding rows of concentrate, and the designated prices were stamped on every can and multipack in the same manner as shown on the placard. These store prices were then changed in keeping with the total design indicated in Table 13.

Nationally advertised label			Private label							Packer label		
6-oz. row	6-oz. row	12-oz. row	6-oz. row	6-oz. row	6-oz. row	12-oz. row	12-oz. row	6-pack row	6-pack row	6-oz. row	6-oz. row	12-oz. row

Figure 3.--Retail display arrangement by label and can size used in the study of demand and substitution relationships for frozen orange concentrate, 1964

Table 14.--Test prices converted to retail prices charged for estimating demand relationships for frozen orange concentrate, 1964

Nationally advertised label			Packer and private label			
Test price 6-oz. can	Retail price 6-oz. can	Retail price 12-oz. can	Test price 6-oz. can	Retail price 6-oz. can	Retail price 12-oz. can	Retail price 6-oz. multipack
			Dollars			
.21	2/.41	.41	.17	2/.33	.33	.99
.23	2/.45	.45	.19	2/.37	.37	1.11
.25	2/.49	.49	.21	2/.41	.41	1.23
.27	2/.53	.53	.23	2/.45	.45	1.35
.29	2/.57	.57	.25	2/.49	.49	1.47
.31	2/.61	.61	.27	2/.53	.53	1.59
.33	2/.65	.65	.29	2/.57	.57	1.71
.35	2/.69	.69	.31	2/.61	.61	1.83
.37	2/.37	.73	.33	2/.65	.65	1.95

The critical nature of pricing and inventory factors required that project personnel maintain complete control over the handling of frozen orange concentrate at all stages in the stores. The store enumerator took over complete management of frozen orange concentrate, including the maintenance of storage inventories in each store, and the ordering of concentrate to replenish these inventories. Orders were phoned in daily to the project headquarters in Dayton, and deliveries were made as needed to all 18 stores from a cold storage warehouse in which large stocks of each brand had been stored specifically for the study. Within each store, the enumerator had to insure that the concentrate displays were adequately stocked at all times, with variability in level for the display bins being allowed between a minimum of 2-can layers and a maximum of the capacity of the freezer.

This over-all management procedure resulted in the maintenance of a strict inventory control process for frozen orange concentrate over all phases of handling during the conduct of the study.

Method of recording sales data.--A complete count of each can size for each brand of frozen orange concentrate was made daily in every supermarket. Timing of this inventory was left to the discretion of each enumerator as to morning or evening hours; however, these counts had to be taken when the stores were closed, not while customers might still be shopping the displays. A standardized series of forms for each label was used for recording these data. This process gave a continuing series of ending and beginning display and storage inventories to which subtraction was applied to obtain daily sales of each brand of frozen orange concentrate.

Store-project relations

The individual supermarket has thus far been regarded in this project as an experimental unit being subjected to control requirements dictated by the research procedure. Within the 18 test stores, it was important that all study controls and activities surrounding the merchandising of frozen orange concentrate be unimpaired. However, neither the stores' operations nor the relationships between the cooperating agency and its customers could be jeopardized by this project. In order to insure that the regular functioning of each store would not be upset by the study, the controls regarding advertising, pricing, display size, supply, collection of data, and the conduct of the enumerators were discussed thoroughly with the executives of the cooperating organization, with a complete understanding concerning these factors being accomplished prior to the beginning of the experiment.

In some stores revenue was lost on each can sold because the test price levels were under the normal price the cooperating agency could charge. A system of compensating for this loss was worked out to the satisfaction of all concerned. A final agreement was made between the project supervisors and the executives of the cooperating firm concerning store performance data in the 18 stores used. While daily customer counts were all that were needed for the analytical procedure, previous experience in similar projects led to the belief that total dollar sales and sales by departments should also be tabulated for each store on a daily basis in the event that such information could be of possible use in the analysis. These data were made available through the office of the comptroller of the firm in such a manner as to be easily converted to the time period and store classifications specified for use in the analytical procedure.

CHAPTER V

ANALYSIS OF THE ALTERNATIVE REGRESSION MODELS

The sales information recorded in this study was analyzed within the framework of each analytical technique. The process of preparing these sales figures for inputs to the three regression procedures, while involving algebraic transformations and adjustments, could not be allowed to alter in any way the basic relationships specified in the economic model. Within this restriction, data manipulations and various regression models were considered as methodological probes designed to ultimately provide a statistically sound and economically consistent system within which an evaluation of consumer responses to varying price situations could be carried on.

Transformations and Adjustments in the Fitting Procedures

Each analytical procedure conformed to the economic model with respect to the nature of the variables and the classification of the data. The quantity of each brand of concentrate sold was specified as a function of the three brand prices. These prices were varied in the test stores within a framework of exogenous factors either assumed to be constant or controlled by design. As long as the direction of the economic relationships was not altered, and the exogenous factors were kept free of price effects, transformations performed on either dependent or independent variables were regarded as noncontradictory to the general assertions of the economic model.

Calculations common to all procedures

The normal routine of store activity resulted in a considerable degree of variation in daily customer traffic over all stores. In connection with this phenomenon, sales of frozen orange concentrate tended to fluctuate widely between individual stores. In terms of sheer volume, the number of customers shopping in a retail store would appear to have a positive effect on sales volumes of all items in that store. However, the effect on individual food items would also be a function of shopping patterns and customer desires with regard to the item in question. Thus, a purchase rate could be said to exist for customers buying one item in individual stores, which when considered over all stores, could be regarded as an average indicator of relative traffic differences among stores. Consequently, expressing this rate in terms of customer traffic would tend to adjust the various purchases for traffic differences between stores. The desirability of expressing the sales variables free of store traffic differences coupled with previous knowledge concerning the usefulness of this technique, resulted in the expression of all unit sales figures as purchase rates.

A summary of the sales information on a period basis was made prior to the specification of transformed unit sales figures. The process used was to divide, within each brand, the total ounces of concentrate sold for a given day in a given store by the corresponding total customer count. The resulting ratio was then multiplied by 100 and added to the ratios of the two remaining days in the respective pricing period. This summation of purchase rates per 100 customers per day into an average purchase rate per 300 customers for a 3-day period was deemed proper in terms of the difference equation analysis used in the regression procedure. Essentially, the difference in purchase rates per 100 customers per day within a base period from purchase rates on matching days could have been used as a dependent variable in the regression equations. However, since the prices of the test brands were set on a 3-day basis, these purchase rate differences would have been regressed on the same price difference. Therefore, summing these rates across the three days and then taking the difference of base and test week summations would result in the total change in purchase rates for one test period being regressed on the same price differential. This new figure resulted in an average purchase rate per 300 customers for each day within each period, with the purchase rate per 100 customers on each day receiving equal weight in the total. The new dependent variables were used in the regression procedure approximating the following generalized economic model:

$$Y_{1-k} = f_k(X_1, X_2, X_3) \tag{5.1}$$

$$Y_{2-k} = f_k(X_1, X_2, X_3) \tag{5.2}$$

$$Y_{3-k} = f_k(X_1, X_2, X_3) \tag{5.3}$$

where:

Y_{i-k}, $i=1,2,3$, $k=2,3,4,5,T$ = the adjustment quantity of the i^{th} brand sold during the k^{th} week, with k being test week 2, 3, 4 and 5 and over-all equation T

f_k = the relationship among the various prices in week equation k and the corresponding sales figures

X_1, X_2, X_3 = the test prices

Model I

The general form of f_k for each brand in analytical Model I was assumed to be:

$$\frac{Y^*_{i-kj}}{Y_{i-kj}} = b_{i0-k}\left(\frac{X^*_1}{X_{1-kj}}\right)^{b_{i1}}\left(\frac{X^*_2}{X_{2-kj}}\right)^{b_{i2}}\left(\frac{X^*_3}{X_{3-kj}}\right)^{b_{i3}} \qquad (5.4)$$

where:

Y^*_{i-kj} = the expected sales of the i^{th} brand during the k^{th} week in the j^{th} store

Y_{i-kj} = the actual sales of the i^{th} brand during the k^{th} week in the j^{th} store

X^*_i = the base or average price level charged for the i^{th} brand

X_{i-kj} = the test price level charged for the i^{th} brand during the k^{th} week in the j^{th} store

$b_{i0}, b_{i1}, b_{i2}, b_{i3}$ = regression coefficients

with subscript ranges the same as given under equation (5.3).

This equation system, derived from the difference equation approach to the store variability problems discussed in Chapter III, involved two basic assertions as to the nature of price-induced sales variation for each brand. The first proposition was that, while the absolute quantities sold of a given brand could vary considerably from store to store at the same price level, the proportionate quantity changes for this same price situation would tend to be equal. In addition, changes in customer responses due to some short-run region disturbance not associated with price variation could be expressed as an average tendency across all stores. Within these concepts and the general difference equation framework, store variability and short-run exogenous influences were compensated for by utilizing expected versus actual sales figures.

Expected sales were defined as the amount of a single brand of concentrate that would have been sold during a test week in a single test store if the average or base price had been charged during that period. This figure was calculated by multiplying the sales of each brand in each store during the base week by an index number based on sales variations at constant prices in the two control stores. Thus, expected sales for brand i in (5.4) were given for each regression in Model I by the following formula:

$$Y^*_{i-kj} = \frac{c}{a} \cdot b \tag{5.5}$$

where:

a = the sum of the sales of brand i in two control stores during the base period

b = the sales of brand i in one of the test stores during the base period

c = the sum of the sales of brand i in two control stores during the test period

Prior to fitting the constants for Model I, equation (5.4) was transformed to a first-difference structure which was linear in natural logarithms. The final regression form was then specified as:

$$\begin{aligned} \ln Y^*_{i-kj} & \\ - \ln Y_{i-kj} &= \ln b_{i0-k} + b_{i1}(\ln X^*_1 - \ln X_{1-kj}) + b_{i2}(\ln X^*_2 - \ln X_{2-kj}) \\ &\quad + b_{i3}(\ln X^*_3 - \ln X_{3-kj}) \end{aligned} \tag{5.6}$$

This completed the adjustments and transformations of sales data for analytical Model I.

Model II

The assertions regarding between-store variation and short-run exogenous influences as indicated in Model I were applicable to the same degree in this procedure although the method of compensating for these factors was considerably revised. The generalized proof of the difference equation technique used in this study was the starting point for development of Model II. In Chapter III, expansion of the arithmetic differences in equation (3.12), coupled with the addition of square and cross-product terms, resulted in equation (3.19). The base sales figure was transferred to the right-hand portion of the equation and subsequently was assumed to be compensated for in the quantity intercept. Thus, the sales in the test weeks were fit against the actual prices charged. The linear regression coefficients were considered a composite of price effects with the differences between stores represented in the regression constant b_{i0}. Within this structure, the generalized form of the equations fit with Model II was specified as:

$$\begin{aligned} Y_{i-kj} &= b_{i0-k} + b_{i1}X_{1-kj} + b_{i2}X_{2-kj} + b_{i3}X_{3-kj} \\ &\quad + b_{i4}(X_{1-kj})^2 + b_{i5}(X_{2-kj})^2 + b_{i6}(X_{3-kj})^2 \\ &\quad + b_{i7}(X_{1-kj})(X_{2-kj}) + b_{i8}(X_{1-kj})(X_{3-kj}) \\ &\quad + b_{i9}(X_{2-kj})(X_{3-kj}) \end{aligned} \tag{5.7}$$

where:

Y_{i-kj} = the actual amount of the i^{th} brand sold during the k^{th} week in the j^{th} store

X_{i-kj} = the test price level charged for the i^{th} brand during the k^{th} week in the j^{th} store

$b_{i0}, b_{i1}, \ldots b_{i9}$ = regression coefficients

with subscript ranges the same as given under equation (5.3).

The dependent variable in this model was adjusted for short-run exogenous factors by an index of variability in sales at constant prices computed from the control store data. The independent variables were transformed to values corresponding to Tramel's triple cube design matrix. This procedure has been shown by Tramel to reduce correlation between pairs of certain coefficients and the variance of quadratic and interaction coefficients.[1] This transformation was performed on each price level in the following manner:

$$X'_{i-kj} = (X_{i-kj} - X^*_i)(.2396) \tag{5.8}$$

where:

X'_{i-kj} = the transformed value of the test price charged for the i^{th} brand during the k^{th} week in the j^{th} store

X_{i-kj} = the test price charged for the i^{th} brand during the k^{th} week in the j^{th} store

X^*_i = the average or base price for the i^{th} brand

The square and cross-product terms in the matrix of independent variables were derived by use of the linear transformed price values.

Model III

Analytical Model III was developed from the same set of assumptions and difference equations as Model I. This development proceeded no farther than acceptance of the basic assertions concerning store variability and exogenous factors and the expansion of equation (3.12) to include quadratic and cross-product terms. A control store index was still used to adjust sales in each store for nonprice influences assumed to be operating across all stores. Moreover, the concept of fitting quantity variations against price variations was also maintained. The difference between this approach and the technique used in Model I was that sales of each brand during the base week in each store were used rather than expected sales in that the control store adjustment was performed on actual sales during each week. No other transformations were made on the data for this technique, thus resulting in the following equation form:

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[1] Tramel, p. 174.

$$\begin{aligned} Y^{*}_{i-j} - Y_{i-kj} = {} & b_{i0-k} + b_{i1}(X^{*}_{1}-X_{1-kj}) + b_{i2}(X^{*}_{2}-X_{2-kj}) \\ & + b_{i3}(X^{*}_{3}-X_{3-kj}) + b_{i4}(X^{*}_{1}-X_{1-kj})^{2} \\ & + b_{i5}(X^{*}_{2}-X_{2-kj})^{2} + b_{i6}(X^{*}_{3}-X_{3-kj})^{2} \\ & + b_{i7}[(X^{*}_{1}-X_{1-kj})(X^{*}_{2}-X_{2-kj})] \\ & + b_{i8}[(X^{*}_{1}-X_{1-kj})(X^{*}_{3}-X_{3-kj})] \\ & + b_{i9}[(X^{*}_{2}-X_{2-kj})(X^{*}_{3}-X_{3-kj})] \end{aligned} \tag{5.9}$$

where:

Y^{*}_{i-j} = the sales of the i^{th} brand during the base period in the j^{th} store

Y_{i-kj} = the sales of the i^{th} brand during the k^{th} week in the j^{th} store

X^{*}_{i} = the base or average price level for the i^{th} brand

X_{i-kj} = the test price level charged for the i^{th} brand during the k^{th} week in the j^{th} store

$b_{i0}, b_{i1}, \ldots b_{i9}$ = regression coefficients

Preliminary Regression Analysis

A stepwise regression program was used in obtaining estimates of the parameters in the three models.[2] The "F" level in the computer routine was set at the minimum value to allow inclusion of all variables in the final step of the program. In all three analytical models, one equation per brand was fit for each of the four test weeks. In addition, one equation was fit again for each brand to the data in the individual model poolled over all test weeks. This resulted in the fitting of 15 regression equations to the sales data within each model.

Initial results for Model I and Model II

The initial regression procedure followed the exact specifications established for each analytical model. Concern over the high degree of variability normally present in economic data prompted a stepwise investigation of the results of fitting each model. Consequently, the first regression for Models I and II resulted in the coefficients listed in Tables 15 and 16. The criterion chosen for analyzing each equation system as to the accountability for variations in sales figures was a judgment examination of the coefficient of multiple correlation for each single regression function. It was felt that since essentially the same independent variables were defined for each process, the coefficients should not have differed greatly from one method to the next; thus making any decision with regard to refitting each equation applicable to all three models. The respective multiple correlation coefficients for these first two analytical models are given in Table 17.

The squared values of the correlation coefficients in Table 17 represent the percentage of the total sums of squares attributable to

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[2] A. S. Goldberger, Econometric Theory (New York: John Wiley & Sons, Inc., 1964), p. 194.

Table 15.--Initial regression coefficients and standard errors calculated using analytical Model I for estimating the demand relationships for frozen orange concentrate, 1964*

X_i**	Week 2 b	Week 2 s_b	Week 3 b	Week 3 s_b	Week 4 b	Week 4 s_b	Week 5 b	Week 5 s_b	Week T# b	Week T# s_b
					Nationally advertised brand					
X_0	0.16182	--	0.35548	--	0.22331	--	0.56772	--	0.32715	--
X_1	-2.11539	0.50699	-1.68620	0.73326	-2.81877	0.62407	-2.16268	0.75573	-2.19575	0.34169
X_2	--	--	0.22491	0.62817	-0.35161	0.53463	0.77790	0.64742	0.15845	0.29272
X_3	0.53719	0.43433	-0.22770	0.62817	0.80360	0.53463	1.42826	0.64742	0.63535	0.29272
					Packer brand					
X_0	0.30446	--	0.29604	--	0.29000	--	0.16333	--	0.26346	--
X_1	0.67054	0.64493	0.66415	0.76388	1.35352	0.79933	0.27964	0.46262	0.74196	0.33187
X_2	-4.50918	0.55250	-3.61926	0.65440	-4.58605	0.68477	-4.35187	0.39632	-4.26659	0.28431
X_3	0.97850	0.55250	1.48470	0.65440	1.05003	0.68477	1.91882	0.39632	1.35801	0.28431
					Private brand					
X_0	0.28580	--	0.16075	--	0.27182	--	0.08079	--	0.19979	--
X_1	1.07288	0.56517	0.99550	0.56257	0.61143	0.41104	1.09132	0.47017	0.94278	0.24736
X_2	0.39028	0.48417	0.49708	0.48195	0.62755	0.35213	0.81456	0.40278	0.58237	0.21191
X_3	-2.59423	0.48417	-2.14825	0.48195	-2.38978	0.35213	-2.51879	0.40278	-2.41276	0.21191

*The dependent variables are Y = ln of expected sales brand 1 - ln of actual sales brand 1 for the nationally advertised brand; Y = ln of expected sales brand 2 - ln of actual sales brand 2 for the packer brand; Y = ln of expected sales brand 3 - ln of actual sales brand 3 for the private brand.

**The independent variables are X_0 = a column of ones in the X matrix
X_1 = ln mean price brand 1 - ln test price brand 1
X_2 = ln mean price brand 2 - ln test price brand 2
X_3 = ln mean price brand 3 - ln test price brand 3

#Week equation T was the equation fit for each brand over all data collected during weeks 2-5.

Table 16.--Initial regression coefficients and standard errors calculated using analytical Model II for estimating the demand relationships for frozen orange concentrate, 1964*

X_i**	Week 2		Week 3		Week 4		Week 5		Week T#	
	$\hat{b}$	$s_{\hat{b}}$	$\hat{b}$	$s_{\hat{b}}$	$\hat{b}$	$s_{\hat{b}}$	$\hat{b}$	$s_{\hat{b}}$	$\hat{b}$	$s_{\hat{b}}$
					Nationally advertised brand					
X_0	28.64843	--	27.33314	--	30.69481	--	23.18054	--	27.46421	--
X_1	- 7.90604	3.39015	- 6.67478	3.95822	- 8.11414	3.36888	- 4.75430	3.12599	- 6.86232	1.53579
X_2	3.29276	3.39015	2.51701	3.95822	2.83495	3.36888	3.45614	3.12599	3.02522	1.53579
X_3	7.19354	3.39015	4.48448	3.95822	7.04766	3.36888	6.08757	3.12599	6.20331	1.53579
X_4	2.19583	3.46347	0.57106	4.04383	- 0.37817	3.44175	- 0.10263	3.19360	0.57153	1.56901
X_5	- 3.51692	3.46347	- 2.60866	4.04383	- 3.42847	3.44175	- 1.48921	3.19360	- 2.76081	1.56901
X_6	- 1.69381	3.46347	- 1.60038	4.04383	- 2.09478	3.44175	- 2.25240	3.19360	- 1.91034	1.56901
X_7	- 2.88698	3.03197	- 4.29376	3.54002	- 1.62632	3.01295	- 2.45973	2.79572	- 2.81670	1.37353
X_8	- 0.55722	3.03197	0.43623	3.54002	0.52087	3.01295	1.53268	2.79572	0.48314	1.37353
X_9	4.11075	3.03197	4.18083	3.54002	3.59161	3.01295	2.62238	2.79572	3.62639	1.37353
					Packer brand					
X_0	43.86249	--	44.32488	--	50.03202	--	46.18232	--	46.10039	--
X_1	3.26925	5.95665	4.14858	5.96062	2.86873	6.62256	0.63538	5.99799	2.73049	2.67836
X_2	- 20.47703	5.95665	- 17.19803	5.96062	-20.61714	6.62256	- 21.61288	5.99799	-19.97627	2.67836
X_3	14.43889	5.95665	15.61812	5.96062	16.46155	6.62256	16.49614	5.99799	15.75367	2.67836
X_4	1.97119	6.08549	- 0.49164	6.08955	- 2.82073	6.76581	1.68807	6.12773	0.08673	2.73629
X_5	- 2.40539	6.08549	- 1.32607	6.08955	- 3.16497	6.76581	- 1.68895	6.12773	- 2.14634	2.73629
X_6	- 0.90760	6.08549	- 0.69971	6.08955	0.66912	6.76581	0.02956	6.12773	- 0.22715	2.73629
X_7	1.00610	5.32732	0.21688	5.33087	0.79908	5.92287	1.45593	5.36429	0.86950	2.39539
X_8	8.60035	5.32732	5.62804	5.33087	4.07550	5.92287	5.78625	5.36429	6.02253	2.39539
X_9	- 7.39435	5.32732	- 7.72251	5.33087	- 7.81424	5.92287	- 6.20904	5.36429	- 7.28504	2.39539
					Private brand					
X_0	101.84324	--	104.73959	--	85.82216	--	105.24748	--	99.41306	--
X_1	- 2.41846	11.98758	1.43831	9.92215	- 2.99863	6.57352	0.69993	9.77497	- 0.81971	4.31300
X_2	- 0.95341	11.98758	- 1.89971	9.92215	- 0.89670	6.57352	- 0.74171	9.77497	- 1.12288	4.31300
X_3	- 16.51232	11.98758	- 15.97369	9.92215	-17.91239	6.57352	- 23.34355	9.77497	-18.43549	4.31300
X_4	- 8.90320	12.24687	- 2.85131	10.13676	- 0.57243	6.71571	- 4.31610	9.98640	- 4.16075	4.40629
X_5	- 5.41681	12.24687	- 8.62936	10.13676	- 9.01873	6.71571	- 7.66427	9.98640	- 7.68228	4.40629
X_6	- 6.18100	12.24687	- 7.65956	10.13676	- 3.30772	6.71571	- 3.44697	9.98640	- 5.14880	4.40629
X_7	- 4.84595	10.72106	- 8.02747	8.87385	- 1.86143	5.87902	5.82672	8.74222	- 2.22703	3.85732
X_8	10.69609	10.72106	9.48210	8.87385	12.95394	5.87902	10.51887	8.74222	10.91275	3.85732
X_9	5.49967	10.72106	6.97607	8.87385	4.85614	5.87902	4.11906	8.74222	5.36274	3.85732

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*The dependent variables are Y = sales of brand I adjusted by a control store index for the nationally advertised brand;

Table 16.--Continued

Y = sales of brand 2 adjusted by a control store index for the packer brand; Y = sales of brand 3 adjusted by a control store index for the private brand.

**The independent variables are X_0 = a column of ones in the X matrix
X_1 = test price of brand 1, modified for Tramel's matrix
X_2 = test price of brand 2, modified for Tramel's matrix
X_3 = test price of brand 3, modified for Tramel's matrix
$X_4 = (X_1)^2$
$X_5 = (X_2)^2$
$X_6 = (X_3)^2$
$X_7 = X_1X_2$
$X_8 = X_1X_3$
$X_9 = X_2X_3$

#Week equation T was the equation fit for each brand over all data collected during weeks 2-5.

Table 17.--Multiple correlation coefficients by brand and week equation for the initial regression using analytical Model I and II in estimating the demand relationships for frozen orange concentrate, 1964

Week equation	Brand		
	National	Packer	Private
		Model I coefficients	
2	.62841	.84646	.73515
3	.40665	.75202	.67963
4	.67219	.80069	.80457
5	.58387	.91510	.79630
T	.52125	.81907	.74160
		Model II coefficients	
2	.63996	.71351	.42593
3	.51878	.68061	.49672
4	.62983	.67855	.64740
5	.56321	.71692	.54157
T	.57078	.69071	.48618

regression on the defined independent variables. In Model I, less than 50 percent of the total variability was accounted for over all weeks for the nationally advertised brand. The total fit over the private brand in this model was also poor, even though it was better than the fit for the national brand. The results for Model II exhibited this same pattern.

The total association between the dependent variable and the independent variables was not important in this study because prediction of sales figures was not desired. It was nonetheless regarded as desirable that the coefficients of determination for the final equations be as large as possible. In consideration of this viewpoint and the results of the initial run in fitting Models I and II, the possibility of redefining each model structure to accommodate variables previously excluded from the analysis was investigated.

Examination of the basic data indicated that a definite pattern existed over periods in the adjusted sales figures used within weeks in both models. The prospect that a larger volume of juice was purchased on the last three days of the week than on the first three days had been considered earlier; however, the use of differences in sales between base periods and test periods was assumed to have eliminated this problem. Thus, the approach was regarded as satisfactory in accommodating period variability in concentrate sales essentially adjusting for differences between stores as well as differences within the same store across periods. The likelihood of the latter assertion breaking down due to a general confounding of period and treatment effects was considered strong enough to support including a means of investigating period effects in the initial run of analytical Model III.

It was assumed that any period effect would be directly related to each experimental unit, and not related in lagged fashion to previous units. This effect would be unique within each week equation for each brand. In addition, this effect was considered to be temporal in nature, and a simple designation of the period source of each observation was considered sufficient to provide a quantitative scale of measure for this factor. Thus, a dummy variable was added to the matrix of independent variables which took on the value zero when the respective observation was drawn from the first three days of the week and the value of one when the last three days were involved.[3] The results of the first Model III regression using this new variable are shown in Table 18. The corresponding multiple correlation coefficients are listed in Table 19.

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[3] Johnston, p. 221.

With few exceptions, there was a definite increase in the multiple correlation coefficients obtained in Model III as compared to the corresponding values obtained for Models I and II. Within the individual regression planes, the size of the period coefficient was larger than the corresponding standard error component for week 3 for the national label, weeks 2, 4, 5 and T for the packer brand, and all weeks for the private brand. Although a formal statistical test was not imposed on these results, all three analytical models were altered to concur with the outcome, and a period variable was included in the final regression equations.

Alteration of the initial fitting procedure

In the process of expanding the equations of each analytical model, the primary concern was to provide a system of variables which would allow efficient estimation of the price effects within the structure of each technique. Rather than changing the basic form of each model, the new variables were introduced into each regression equation in the same manner as the linear price effects, disregarding the quadratic and interaction terms.

At this point in the regression process a second term was considered for inclusion in each equation. This factor was used to depict the total volume of business in each store as measured by total dollar sales. The problem of analysis across all stores which resulted in calculating the dependent variable in ounces per 300 customers was

Table 18.--Initial regression coefficients and standard errors calculated using analytical Model III for estimating the demand relationships for frozen orange concentrate, 1964*

X_i**	Week 2		Week 3		Week 4		Week 5		Week T#	
	$\hat{b}$	$s_{\hat{b}}$	$\hat{b}$	$s_{\hat{b}}$	$\hat{b}$	$s_{\hat{b}}$	$\hat{b}$	$s_{\hat{b}}$	$\hat{b}$	$s_{\hat{b}}$
					Nationally advertised brand					
X_0	0.27845	--	2.64570	--	- 2.02980	--	- 1.69955	--	- 0.20130	--
X_1	- 1.93281	0.53164	- 1.60819	0.72199	- 2.03798	0.63922	- 1.29442	0.44668	- 1.71835	0.26149
X_2	0.14571	0.53164	0.02870	0.72199	0.08231	0.63922	0.38038	0.44668	0.15928	0.26149
X_3	0.13072	0.54445	- 0.54163	0.73938	0.05231	0.65461	0.14270	0.45744	- 0.05397	0.26779
X_4	- 0.15632	0.13092	- 0.09483	0.17780	- 0.03229	0.15741	- 0.04739	0.11000	- 0.08271	0.06440
X_5	0.02216	0.13212	0.05598	0.17943	0.06898	0.15886	0.00120	0.11101	0.03708	0.06499
X_6	0.13433	0.13092	0.08425	0.17780	0.11921	0.15741	0.10713	0.11000	0.11123	0.06440
X_7	- 0.01307	0.11388	0.09774	0.15465	- 0.05203	0.13692	0.05142	0.09568	0.02101	0.05601
X_8	0.18114	0.11690	0.16951	0.15875	0.17608	0.14055	0.11602	0.09822	0.16069	0.05750
X_9	0.03523	0.11472	- 0.05078	0.15580	0.00432	0.13794	- 0.01445	0.09639	- 0.00642	0.05643
X_{10}	0.98115	4.87971	- 9.90147	6.62681	- 1.45235	5.86706	1.85212	4.09990	- 2.13014	2.40010
					Packer brand					
X_0	8.20632	--	5.78829	--	12.42666	--	7.80677	--	8.55701	--
X_1	0.62328	0.79080	0.82005	0.83019	0.47562	0.92250	- 0.08611	0.79309	0.45821	0.37773
X_2	- 5.11375	0.79080	- 4.33281	0.83019	- 5.19324	0.92250	- 5.39060	0.79309	- 5.00760	0.37773
X_3	1.19983	0.80985	1.50349	0.85018	1.20100	0.94472	1.56618	0.81219	1.36762	0.38683
X_4	- 0.19479	0.19474	- 0.06736	0.20444	0.05846	0.22718	- 0.17381	0.19531	- 0.09437	0.09302
X_5	- 0.16694	0.19653	- 0.16772	0.20632	- 0.17022	0.22926	- 0.25568	0.19710	- 0.19014	0.09387
X_6	- 0.08892	0.19474	- 0.10627	0.20444	- 0.21067	0.22718	- 0.14062	0.19531	- 0.13662	0.09302
X_7	0.04422	0.16939	0.07729	0.17782	0.07413	0.19760	0.02855	0.16988	0.05605	0.08091
X_8	- 0.03817	0.17388	0.13051	0.18254	0.31943	0.20284	0.14554	0.17438	0.13933	0.08306
X_9	0.66726	0.17065	0.64485	0.17915	0.73121	0.19907	0.63438	0.17114	0.66942	0.08151
X_{10}	10.94273	7.25838	5.40790	7.61990	12.20840	8.46724	18.30542	7.27935	11.71611	3.46702
					Private brand					
X_0	- 0.38444	--	- 2.02404	--	3.47450	--	- 9.42646	--	- 2.09012	--
X_1	0.13675	1.13187	1.38993	1.13009	0.42840	1.18363	1.13395	1.18793	0.77226	0.56056
X_2	0.89677	1.13187	0.56753	1.13009	0.95518	1.18363	1.08744	1.18793	0.87673	0.56056
X_3	- 6.95629	1.15914	- 6.43936	1.15731	- 5.91351	1.21214	- 7.76268	1.21655	- 6.76796	0.57406
X_4	0.43110	0.27874	0.10735	0.27830	0.10519	0.29148	0.28079	0.29254	0.23111	0.13804
X_5	0.10839	0.28129	0.37334	0.28085	0.24350	0.29416	0.13126	0.29523	0.21412	0.13931
X_6	0.24588	0.27874	0.35180	0.27830	0.22726	0.29148	0.19663	0.29254	0.25539	0.13804
X_7	- 0.11676	0.24244	0.10781	0.24206	- 0.23411	0.25353	- 0.72283	0.25445	- 0.24147	0.12007
X_8	0.18299	0.24888	0.20269	0.24848	- 0.18807	0.26026	0.05337	0.26120	0.06275	0.12326
X_9	0.05715	0.24425	0.04136	0.24386	0.33950	0.25542	0.32999	0.25634	0.19200	0.12096
X_{10}	17.42808	10.38893	-10.79661	10.37252	13.01395	10.86395	27.19734	10.90347	11.71069	5.14510

*The dependent variables are Y = sales of brand 1 during the base week - sales of brand 1 during the test week adjusted by a

Table 18.--Continued

control store index for the nationally advertised brand; Y = sales of brand 2 during the base week - sales of brand 2 during the test week adjusted by a control store index for the packer brand; Y = sales of brand 3 during the base week - sales of brand 3 during the test week adjusted by a control store index for the private brand.

**The independent variables are X_0 = a column of ones in the X matrix
X_1 = mean price of brand 1 - test price of brand 1
X_2 = mean price of brand 2 - test price of brand 2
X_3 = mean price of brand 3 - test price of brand 3
$X_4 = (X_1)^2$
$X_5 = (X_2)^2$
$X_6 = (X_3)^2$
$X_7 = X_1X_2$
$X_8 = X_1X_3$
$X_9 = X_2X_3$
X_{10} = zero for period 1, one for period 2

#Week equation T was the equation fit for each brand over all data collected during weeks 2-5.

Table 19.--Multiple correlation coefficients by brand and week equation for the initial regression using analytical Model III in estimating the demand relationships for frozen orange concentrate, 1964

Week equation	Brand		
	National	Packer	Private
2	.68235	.86798	.84312
3	.54989	.83088	.80466
4	.61694	.84788	.78897
5	.60845	.88302	.87746
T	.57080	.83990	.78498

assumed to be applicable to this sales volume figure, thus resulting in classification of total dollar sales in terms of customer traffic flows in each store. The possibility of average total dollar sales per 300 customers attributing to a significant reduction in the sum of squared residuals in each equation was considered reasonable because of the high degree of variability in concentrate sales between stores and within stores from period to period. A dummy variable approach was not feasible in this case. Therefore, the actual calculated figures for each period were used as the respective values in the matrix of independent variables.

In addition to using a dummy variable to handle period effects, the applicability of this technique for handling temporal shifts in economic data resulted in the utilization of a zero--one variable in investigating shifts in the level of concentrate sales over time. This procedure entailed the addition of a variable to the total equation over all weeks for weeks 3, 4 and 5. The final regression equations were then established in the following manner.

Model I

The equation fit in this model for each individual week was:

$$\begin{aligned}\ln Y^{*}_{i-kj} - \ln Y_{i-kj} &= \ln b_{i0-k} + b_{i1}(\ln X^{*}_{1} - \ln X_{1-kj}) + b_{i2}(\ln X^{*}_{2} - \ln X_{2-kj}) \\ &\quad + b_{i3}(\ln X^{*}_{3} - \ln X_{3-kj}) + b_{i4}X_{4-k} + b_{i5}X_{5-kj} \qquad (5.10)\end{aligned}$$

where all variables with the exception of X_{4-k}, X_{5-kj} were defined as in (5.6) and:

X_{4-k} = a variable taking on the value zero for period 1 and one for period 2

X_{5-kj} = total dollar sales per 300 customers during the test period in the k^{th} week in the j^{th} store

The total equation fit in this model over all weeks was identical to (5.10) with addition of the following dummy variables and corresponding regression coefficients:

X_6 = the value one for observations from week 3, zero for all other weeks

X_7 = the value one for observations from week 4, zero for all other weeks

X_8 = the value one for observations from week 5, zero for all other weeks

Model II

The equation fit for each individual week in this model was:

$$\begin{aligned}Y_{i-kj} &= b_{i0-k} + b_{i1}X_{1-kj} + b_{i2}X_{2-kj} + b_{i3}X_{3-kj} \\ &\quad + b_{i4}(X_{1-kj})^{2} + b_{i5}(X_{2-kj})^{2} + b_{i6}(X_{3-kj})^{2} \\ &\quad + b_{i7}(X_{1-kj})(X_{2-kj}) + b_{i8}(X_{1-kj})(X_{2-kj}) \\ &\quad + b_{i9}(X_{2-kj})(X_{3-kj}) + b_{i-10}X_{10-k} \\ &\quad + b_{i-11}X_{11-kj} \qquad (5.11)\end{aligned}$$

where all variables except X_{10-k}, X_{11-kj} were defined as in (5.7) and:

X_{10-k} = a variable taking on the value zero for period 1 and one for period 2

X_{11-kj} = total dollar sales per 300 customers during the test period in the k^{th} week and j^{th} store

Equation (5.11) was fit over all weeks in this model with the following dummy variables added:

X_{12} = the value one for observations from week 3, zero for all other weeks

X_{13} = the value one for observations from week 4, zero for all other weeks

X_{14} = the value one for observations from week 5, zero for all other weeks

Model III

The equation fit in analytical Model III for each week was:

$$\begin{aligned}
Y^{*}_{i-j} - Y_{i-kj} = {} & b_{i0-k} + b_{i1}(X^{*}_{1}-X_{1-kj}) + b_{i2}(X^{*}_{2}-X_{2-kj}) \\
& + b_{i3}(X^{*}_{3}-X_{3-kj}) + b_{i4}(X^{*}_{1}-X_{1-kj})^{2} \\
& + b_{i5}(X^{*}_{2}-X_{2-kj})^{2} - b_{i6}(X^{*}_{3}-X_{3-kj})^{2} \\
& + b_{i7}[(X^{*}_{1}-X_{1-kj})(X^{*}_{2}-X_{2-kj})] \\
& + b_{i8}[(X^{*}_{1}-X_{1-kj})(X^{*}_{3}-X_{3-kj})] \\
& + b_{i9}[(X^{*}_{2}-X_{2-kj})(X^{*}_{3}-X_{3-kj})] \\
& + b_{i-10}X_{10-k} + b_{i-11}X_{11-kj} \qquad (5.12)
\end{aligned}$$

where all varibles except X_{10-k}, X_{11-kj} were defined as (5.9) and:

X_{10-k} = a variable taking on the value zero for period 1 and one for period 2

X_{11-kj} = total dollar sales per 300 customers during the test period in the k^{th} week and j^{th} store

Equation (5.12) was fit over all weeks in this model with the following dummy variables added:

X_{12} = the value one for observations from week 3, zero for all other weeks

X_{13} = the value one for observations from week 4, zero for all other weeks

X_{14} = the value one for observations from week 5, zero for all other weeks

The Final Estimating Equations

The coefficients resulting from the expanded analytical models are shown in Tables 20, 21 and 22, and the corresponding multiple correlation coefficients are indicated in Table 23. There was a substantial increase in these correlation coefficients over those given in the initial regression for Models I and II. The increase was relatively slight for Model III since the initial run for this technique included the period variable in the equations. The regression coefficients in these functions were utilized in the evaluation of the economic model proposed in this study; however, it was decided to select only one of the three forms for interpretation in the final analysis. The procedure for making this choice evolved from certain statistical considerations, finally culminating in requirements for economic consistency and interpretation. The initial selection process was made within each model by testing for the price-carryover component over all weeks. Following the results of this analysis, the significant regression coefficients in each equation were specified and the elasticity coefficients were derived and interpreted. Finally, the elasticity values to be used in evaluating the economic model were selected with regard to a set of established consistency requirements.

Table 20.--Regression coefficients and standard errors calculated using analytical Model I for estimating the demand relationships for frozen orange concentrate, 1964*

x_i**	Week 2		Week 3		Week 4		Week 5		Week T#	
	b	s_b	$\hat{b}$	$s_{\hat{b}}$	$\hat{b}$	s_b	b	s_b	b	s_b
					Nationally advertised brand					
x_0	2.38812	--	5.07131	--	-0.55593	--	1.57517	--	2.08996	--
x_1	-2.22115†	0.53222	-1.97451†	0.64089	-2.71399†	0.61834	-2.16054†	0.67709	-2.28077†	0.31229
x_2	-0.06693	0.44419	-0.01965	0.53654	-0.34084	0.51214	0.70220†	0.56771	0.06414	0.26042
x_3	0.35156	0.46111	-0.72786†	0.53907	0.57918†	0.52272	0.97949†	0.56533	0.28769†	0.26454
x_4	-0.00635	0.23960	-0.24241	0.29395	-0.50981	0.29738	-0.65265†	0.29841	-0.33810†	0.14234
x_5	-0.31081	0.34201	-0.62934†	0.36808	0.14407†	0.37376	-0.09325	0.41677	-0.24584†	0.18849
x_6	--	--	--	--	--	--	--	--	0.22504†	0.12840
x_7	--	--	--	--	--	--	--	--	0.06603	0.12604
x_8	--	--	--	--	--	--	--	--	0.45086†	0.12661
					Packer brand					
x_0	2.12722	--	0.52122	--	0.44725	--	4.45689	--	-0.13578	--
x_1	0.60064	0.65596	0.71111	0.73714	1.40659†	0.77873	-0.02587†	0.45394	0.80669†	0.31035
x_2	-4.55832†	0.54746	-3.65119†	0.61711	-4.61660†	0.64498	-4.57442†	0.38061	-4.37062†	0.26913
x_3	0.72761†	0.56831	1.13243†	0.62002	0.69463†	0.65831	1.71386	0.37901	1.17141†	0.27424
x_4	-0.17690	0.29531	-0.57914†	0.33809	-0.59620†	0.37452	0.14801	0.20006	-0.39606†	0.09447
x_5	-0.24227	0.42153	0.00956	0.42336	0.02030	0.47071	-0.60348†	0.27942	0.07555†	0.04316
x_6	--	--	--	--	--	--	--	--	0.00440	0.13045
x_7	--	--	--	--	--	--	--	--	-0.00080	0.13043
x_8	--	--	--	--	--	--	--	--	-0.10704	0.13099
					Private brand					
x_0	1.97593	--	1.46373	--	-0.19422	--	0.26862	--	0.83221	--
x_1	1.03392†	0.48438	0.93855†	0.55504	0.63956†	0.44663	1.08487	0.51380	0.92884†	0.24853
x_2	0.33147	0.40427	0.42194	0.46466	0.64719†	0.36993	0.80263†	0.43080	0.55157†	0.20726
x_3	-2.97976†	0.41966	-2.40922†	0.46685	-2.33754†	0.37757	-2.56529†	0.42899	-2.56857†	0.21053
x_4	-0.42955	0.21807	-0.27917†	0.25457	0.02041	0.21480	-0.05792	0.22645	-0.19460†	0.11328
x_5	-0.20574	0.31127	-0.15908	0.31877	0.06319	0.26997	-0.02187	0.31626	-0.06309	0.15001
x_6	--	--	--	--	--	--	--	--	-0.10879†	0.10219
x_7	--	--	--	--	--	--	--	--	-0.00901	0.10031
x_8	--	--	--	--	--	--	--	--	-0.19208†	0.10076

*The dependent variables are Y = ln of expected sales brand 1 - ln of actual sales brand 1 for the nationally advertised brand;

Table 20.--Continued

Y = ln of expected sales brand 2 - ln of actual sales brand 2 for the packer brand; Y = ln of expected sales brand 3 - ln of actual sales brand 3 for the private brand.

[**] The independent variables are X_0 = a column of ones in the X matrix
X_1 = ln mean price brand 1 - ln test price brand 1
X_2 = ln mean price brand 2 - ln test price brand 2
X_3 = ln mean price brand 3 - ln test price brand 3
X_4 = zero for period 1, one for period 2
X_5 = ln total dollar sales per 300 customers
X_6 = one for week 3, zero for all other weeks
X_7 = one for week 4, zero for all other weeks
X_8 = one for week 5, zero for all other weeks

[#] Independent variables X_6, X_7, X_8 were introduced only for week equation T, which was the equation fit for each brand over all data collected during weeks 2-5.

[≠] These coefficients are regarded as significantly different from zero as defined in the test criterion.

Table 21.--Regression coefficients and standard errors calculated using analytical Model II for estimating the demand relationships for frozen orange concentrate, 1964*

x_i**	Week 2		Week 3		Week 4		Week 5		Week T#	
	b	$s_{\hat{b}}$	$\hat{b}$	$s_{\hat{b}}$	$\hat{b}$	s_b	b	s_b	b	s_b
					Nationally advertised brand					
x_0	-15.02986	--	-10.88261	--	3.58891	--	- 5.67122	--	- 1.61110	--
x_1	-10.12304/	3.04639	- 8.82848/	3.57141	- 9.50183/	3.18030	- 6.22293/	2.87440	- 8.72940	1.37321
x_2	2.59013	2.93335	0.36260	3.48093	1.88156	2.99731	1.92367	2.77563	1.58443	1.32042
x_3	4.73028/	3.04917	1.87836	3.44187	4.67691/	3.06164	4.01531/	2.71635	3.88279	1.33403
x_4	1.97643	3.01778	0.67183	3.47248	0.24486	3.04177	0.42042	2.72777	0.84338	1.33434
x_5	- 1.31830	3.21505	- 1.13437	3.81449	- 2.88286	3.37003	- 1.09642	2.98644	- 1.57223	1.44863
x_6	0.96513	3.10939	0.59571	3.49002	- 0.27897	3.10693	- 0.25648	2.81023	0.22386	1.35976
x_7	- 1.92716	2.63640	- 3.82377/	2.99708	- 1.59805	2.64046	- 2.37575/	2.36257	- 2.52593	1.15654
x_8	2.92034	3.10570	2.82252	3.75761	1.18035	3.16198	2.13226	2.82630	2.22431	1.38285
x_9	2.04098	2.78755	1.88080	3.44920	2.71763	2.93536	1.59977	2.68071	2.00245	1.27989
x_{10}	-19.26018/	12.03957	- 6.60443	15.26037	0.16768	13.71414	1.83089	11.54420	- 5.98210	5.55054
x_{11}	0.03621/	0.01309	0.02391/	0.01225	0.01659/	0.01199	0.01703/	0.01123	0.02288	0.00507
x_{12}	--	--	--	--	--	--	--	--	- 7.31145	3.89933
x_{13}	--	--	--	--	--	--	--	--	- 3.30470	3.75173
x_{14}	--	--	--	--	--	--	--	--	- 9.16852	3.76787
					Packer brand					
x_0	-27.81379	--	11.80691	--	10.61602	--	-13.33758	--	- 0.42532	--
x_1	0.58404	4.66858	3.73331	5.01737	2.08507	5.60101	- 2.36023	5.23626	1.05486	2.21711
x_2	-21.77039/	4.49534	-18.54619/	4.89026	-21.88544/	5.27874	-24.76323/	5.05633	-21.84101/	2.13188
x_3	8.73097/	4.67283	11.28729/	4.83538	11.35503/	5.39203	12.16838/	4.94835	11.01120/	2.15385
x_4	2.90648	4.62472	1.26306	4.87838	- 0.68483	5.35704	2.80331	4.96914	1.56346	2.15436
x_5	- 1.32931	4.92704	- 3.38798	5.35886	- 5.15981	5.93515	- 0.95309	5.44036	- 2.71873/	2.33888
x_6	3.94999	4.76512	1.91266	4.90303	3.85051	5.47179	4.16109	5.11935	3.24282/	2.19539
x_7	2.44280	4.04027	0.58798	4.21051	0.98969	4.65027	1.63250	4.30387	1.30438	1.86728
x_8	11.15056/	4.75946	3.56644	5.27896	1.83439	5.56874	6.93598/	5.14864	5.78344/	2.23268
x_9	- 9.05616/	4.27191	- 7.05542/	4.84567	- 7.08780/	5.16964	- 8.26190/	4.88341	- 7.90997/	2.06645
x_{10}	- 3.06778	18.45058	29.47333/	21.43884	30.00204/	24.15276	4.57705	21.02993	15.86466/	8.96160
x_{11}	0.04935/	0.02006	0.00979	0.01721	0.01454	0.02112	0.03487/	0.02045	0.02449/	0.00818
x_{12}	--	--	--	--	--	--	--	--	- 6.48090/	6.29564
x_{13}	--	--	--	--	--	--	--	--	- 0.47252	6.05734
x_{14}	--	--	--	--	--	--	--	--	0.69255	6.08341

Table 21.--Continued

X_i[**]	Week 2		Week 3		Week 4		Week 5		Week T[#]	
	b	s_b	b	s_b	b	s_b	b	s_b	b	s_b
					Private brand					
X_0	-58.69185	--	24.04754	--	25.53406	--	12.60711	--	14.87879	--
X_1	- 9.06999	9.30798	- 1.87868	8.45865	- 6.89096≁	6.28428	- 5.11899	9.78247	- 5.43959≁	3.85589
X_2	- 3.75631	8.96258	- 6.02761	8.24435	- 3.09451	5.92479	- 6.02034	9.44632	- 4.87818≁	3.70766
X_3	-28.18229≁	9.31645	-23.31121≁	8.15183	-22.10320≁	6.05151	-28.28908≁	9.24458	-25.06252≁	3.74587
X_4	- 7.67484	9.22054	- 1.18957	8.22433	--	--	- 3.80838	9.28343	- 3.07784	3.74676
X_5	- 1.31312	9.82329	- 8.39568	9.03436	- 5.98355	6.52806	- 3.99284	10.16376	- 5.28673≁	4.06766
X_6	4.36806	9.50044	- 2.37686	8.26589	0.38031	6.05411	2.50714	9.56406	- 0.64937	3.81811
X_7	- 1.53558	8.05529	- 7.06011	7.09838	- 1.89611	5.21954	5.98845	8.04056	- 1.44968	3.24748
X_8	18.51923≁	9.48917	10.96798≁	8.89966	16.50935≁	6.24074	15.26545≁	9.61877	14.62149≁	3.88297
X_9	0.61738	8.51712	4.39790	8.16920	1.60853	5.79780	- 1.00151	9.12326	1.62151	3.59387
X_{10}	-25.96032	36.78581	16.52235	36.14317	-19.04771	26.94051	-20.01687	39.28846	- 7.73597	15.58557
X_{11}	0.11726≁	0.03999	0.04131≁	0.02902	0.04315≁	0.02366	0.06296≁	0.03821	0.05794≁	0.01423
X_{12}	--	--	--	--	--	--	--	--	- 9.24895	10.94908
X_{13}	--	--	--	--	--	--	--	--	-14.16170≁	10.53462
X_{14}	--	--	--	--	--	--	--	--	1.44736	10.57997

[*] The dependent variables are Y = sales of brand 1 adjusted by a control store index for the nationally advertised brand; Y = sales of brand 2 adjusted by a control store index for the packer brand; Y = sales of brand 3 adjusted by a control store index for the private brand.

[**] The independent variables are X_0 = a column of ones in the X matrix
X_1 = test price of brand 1, modified for Tramel's matrix
X_2 = test price of brand 2, modified for Tramel's matrix
X_3 = test price of brand 3, modified for Tramel's matrix
X_4 = $(X_1)^2$
X_5 = $(X_2)^2$
X_6 = $(X_3)^2$
X_7 = X_1X_2
X_8 = X_1X_3
X_9 = X_2X_3
X_{10} = zero for period 1, one for period 2
X_{11} = total dollar sales per 300 customers
X_{12} = one for week 3, zero for all other weeks
X_{13} = one for week 4, zero for all other weeks
X_{14} = one for week 5, zero for all other weeks

[#] Independent variables X_{12}, X_{13}, X_{14} were introduced only for week equation T, which was the equation fit for each brand over all data collected during weeks 2-5.

≁ These coefficients are regarded as significantly different from zero as defined in the test criterion.

Table 22.--Regression coefficients and standard errors calculated using analytical Model III for estimating the demand relationships for frozen orange concentrate, 1964*

X_i**	Week 2 $\hat{b}$	Week 2 $s_{\hat{b}}$	Week 3 $\hat{b}$	Week 3 $s_{\hat{b}}$	Week 4 $\hat{b}$	Week 4 $s_{\hat{b}}$	Week 5 $\hat{b}$	Week 5 $s_{\hat{b}}$	Week T# b	Week T# s_b
					Nationally advertised brand					
X_0	24.51786	--	17.06894	--	4.77197	--	1.20243	--	12.40108	--
X_1	- 2.28201/	0.49295	- 1.90887/	0.76585	- 2.18366/	0.69990	- 1.35415/	0.49668	- 1.97188/	0.27293
X_2	0.06030	0.47466	- 0.20237	0.74645	0.01903	0.65963	0.33556	0.47961	0.01201	0.26244
X_3	- 0.10169	0.49340	- 0.61937	0.73807	- 0.00654	0.67379	0.13043	0.46937	- 0.14401	0.26514
X_4	- 0.13160/	0.11700	- 0.06713	0.17842	- 0.02640	0.16039	- 0.04422	0.11293	- 0.06656/	0.06354
X_5	- 0.08253	0.12465	- 0.03534	0.19599	0.02743	0.17770	- 0.01380	0.12364	- 0.03482	0.06899
X_6	0.05638	0.12055	0.04998	0.17932	0.09958	0.16383	0.09801	0.11635	0.07331/	0.06475
X_7	- 0.04555	0.10222	0.08704	0.15399	- 0.05063	0.13923	0.05151	0.09781	0.01508	0.05508
X_8	0.02717	0.12041	0.04453	0.19307	0.12791	0.16673	0.09764	0.11701	0.06751/	0.06585
X_9	0.12491/	0.10808	0.04604	0.17722	0.04104	0.15478	0.00116	0.11098	0.06329/	0.06095
X_{10}	18.47118/	8.13097	3.53686	13.65791	4.76968	12.59655	4.05144	8.32535	8.75846/	4.60432
X_{11}	- 0.02250/	0.00884	- 0.01232/	0.01097	- 0.00618	0.01102	- 0.00248	0.00810	- 0.01157/	0.00420
X_{12}	--	--	--	--	--	--	--	--	0.44670	3.23460
X_{13}	--	--	--	--	--	--	--	--	0.47556	3.11216
X_{14}	--	--	--	--	--	--	--	--	0.98939	3.12556
					Packer brand					
X_0	45.85845	--	0.77689	--	14.33285	--	23.76123	--	17.24950	--
X_1	0.08085	0.72242	0.92452/	0.90551	0.43479	1.01770	- 0.41447	0.86396	0.24318	0.39404
X_2	- 5.24643/	0.69561	- 4.25252/	0.88257	- 5.21097/	0.95915	- 5.63705/	0.83428	- 5.13250/	0.37889
X_3	0.83881/	0.72307	1.53050/	0.87266	1.18450/	0.97973	1.49873/	0.81646	1.29126/	0.38280
X_4	- 0.15640	0.17146	- 0.07699	0.21095	0.06011	0.23322	- 0.15638	0.19645	- 0.08068	0.09174
X_5	- 0.32956/	0.18267	- 0.13599	0.23173	- 0.18187	0.25839	- 0.33816/	0.21507	- 0.25112/	0.09960
X_6	- 0.21000/	0.17667	- 0.09437	0.21202	- 0.21617	0.23822	- 0.19077	0.20238	- 0.16878/	0.09349
X_7	- 0.00622	0.14980	0.08101	0.18207	0.07453	0.20245	0.02902	0.17015	0.05102	0.07952
X_8	- 0.27735	0.17646	0.17393	0.22827	0.30593/	0.24244	0.04454	0.20354	0.06030	0.09508
X_9	0.80655	0.15838	0.61120/	0.20954	0.74150/	0.22506	0.72021/	0.19306	0.72855/	0.08800
X_{10}	38.11076/	11.91588	0.73872	16.14844	13.95211	18.31616	30.39684/	14.48190	20.95116/	6.64741
X_{11}	- 0.03495/	0.01295	0.00428	0.01296	- 0.00173	0.01602	- 0.01361	0.01408	- 0.00981/	0.00607
X_{12}	--	--	--	--	--	--	--	--	- 0.92252	4.66989
X_{13}	--	--	--	--	--	--	--	--	8.17691/	4.49312
X_{14}	--	--	--	--	--	--	--	--	2.35127	4.51246

Table 22.--Continued

X_i**	Week 2 $\hat{b}$	Week 2 $s_{\hat{b}}$	Week 3 $\hat{b}$	Week 3 $s_{\hat{b}}$	Week 4 $\hat{b}$	Week 4 $s_{\hat{b}}$	Week 5 $\hat{b}$	Week 5 $s_{\hat{b}}$	Week T# $\hat{b}$	Week T# $s_{\hat{b}}$
					Private brand					
X_0	12.08223	--	-16.15600	--	-10.76653	--	-22.91393	--	2.93430	--
X_1	- 0.04285	1.19889	1.68454/	1.22152	0.73329	1.18635	1.41153/	1.31452	0.79526/	0.59055
X_2	0.85284	1.15441	0.79393	1.19057	1.08765	1.17832	1.29578/	1.26936	0.89009/	0.56785
X_3	- 7.07582/	1.19999	- 6.36319/	1.17721	- 5.79010/	1.21410	- 7.70567/	1.24225	- 6.75979/	0.57370
X_4	0.44381/	0.28456	0.08021	0.28457	0.09289	0.28739	0.26605	0.29889	0.22964/	0.13749
X_5	0.05455	0.30316	0.46282/	0.31260	0.33043/	0.28727	0.20099	0.32724	0.22064/	0.14927
X_6	0.20579	0.29319	0.38537/	0.28601	0.26836	0.29267	0.23903	0.30793	0.25883/	0.14011
X_7	- 0.13346	0.24860	0.11830	0.24561	- 0.23703	0.25098	- 0.72322/	0.25888	- 0.24093/	0.11917
X_8	0.10380	0.29285	0.32514/	0.30793	- 0.08729	0.25193	0.13875	0.30969	0.07120	0.14249
X_9	0.10327	0.26285	- 0.05351	0.28266	0.26266/	0.25261	0.25744	0.29374	0.18568/	0.13188
X_{10}	26.42345/	19.77511	-23.96355/	21.78398	--	--	16.97558	22.03427	10.72284/	9.96249
X_{11}	- 0.01157	0.02150	0.01207	0.01749	0.01293/	0.00941	0.01151	0.02143	0.00105	0.00909
X_{12}	--	--	--	--	--	--	--	--	-15.14950/	6.99878
X_{13}	--	--	--	--	--	--	--	--	- 2.22643	6.73386
X_{14}	--	--	--	--	--	--	--	--	- 7.46850/	6.76284

*The dependent variables are Y = sales of brand 1 during the base week - sales of brand 1 during the test week adjusted by a control store index for the nationally advertised brand; Y = sales of brand 2 during the base week - sales of brand 2 during the test week adjusted by a control store index for the packer brand; Y = sales of brand 3 during the base week - sales of brand 3 during the test week adjusted by a control store index for the private brand.

**The independent variables are X_0 = a column of ones in the X matrix
X_1 = mean price of brand 1 - test price of brand 1
X_2 = mean price of brand 2 - test price of brand 2
X_3 = mean price of brand 3 - test price of brand 3
X_4 = $(X_1)^2$
X_5 = $(X_2)^2$
X_6 = $(X_3)^2$
X_7 = X_1X_2
X_8 = X_1X_3
X_9 = X_2X_3
X_{10} = zero for period 1, one for period 2
X_{11} = total dollar sales per 300 customers
X_{12} = one for week 2, zero for all other weeks
X_{13} = one for week 3, zero for all other weeks
X_{14} = one for week 4, zero for all other weeks

#Independent variables X_{12}, X_{13}, X_{14} were introduced only for week equation T, which was the equation fit for each brand over all data collected during weeks 2-5.

/These coefficients are regarded as significantly different from zero as defined in the test criterion.

Table 23.--Multiple correlation coefficients by brand and week equation for the final regression for analytical Models I, II and III used in estimating the demand relationships for frozen orange concentrate, 1964

Week equation	Brand		
	National	Packer	Private
		Model I coefficients	
2	.66194	.86260	.83990
3	.68069	.81208	.74777
4	.74093	.84529	.80684
5	.74888	.93276	.79828
T	.68354	.85150	.77599
		Model II coefficients	
2	.77495	.86450	.76584
3	.72503	.83475	.75072
4	.76080	.83533	.75045
5	.74619	.84595	.67575
T	.73691	.83440	.69467
		Model III coefficients	
2	.77223	.90509	.84557
3	.58632	.83188	.80955
4	.62466	.84798	.79372
5	.61086	.88857	.87932
T	.60940	.85184	.79712

Statistical Analysis of the Regression Equations

Price-carryover tests

The first consideration in the statistical analysis was the possibility of a change in consumer reactions to the same price over the four weeks of test pricing. The general hypothesis established for testing price-carryover effects was that one regression equation would suffice over all observations. Using the term B_{iK} to denote the vector of regression coefficients associated with week K, this hypothesis was stated formally for one brand as:

$$H_1 : B_{i1} = B_{i2} = B_{i3} = B_{i4} = B_{iT}$$

The test statistic for this hypothesis was:[4]

$$\frac{\frac{S_2}{(N-p)} - (N-P')}{\frac{S_2}{(N-P)}} = F\Big|^{d.f.\ S_3}_{d.f.\ S_2} \qquad (5.13)$$

The sum of squared residuals for S_2 and S_3, along with their respective degrees of freedom, were calculated from Table 24. The critical region for this test was the value $F\Big|^{d.f.\ S_3}_{d.f.\ S_2} > F_{.95}$ with the respective degrees of freedom.

The price-carryover tests for analytical Model I, using (5.13) and the respective values calculated in Table 25, were:

Nationally advertised brand

$$\frac{\frac{11.11144}{15}}{\frac{25.69378}{104}} = 2.99842 \text{ with d.f.}\Big|^{15}_{104}$$

Packer brand

$$\frac{\frac{4.17330}{15}}{\frac{30.02686}{104}} = .96366 \text{ with d.f.}\Big|^{15}_{104}$$

Private brand

$$\frac{\frac{3.84570}{15}}{\frac{16.88587}{104}} = 1.57908 \text{ with d.f.}\Big|^{15}_{104}$$

Similarly, using the data from Table 26, the calculated values for analytical Model II were:

Nationally advertised brand

$$\frac{\frac{1521.56417}{33}}{\frac{23429.37244}{80}} = .15743 \text{ with d.f.}\Big|^{33}_{80}$$

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[4] Ibid., p. 137.

Table 24.--Formulas for calculation of the values in equation (5.6) for one brand of concentrate, 1964

Source of residual sum of squares	Degrees of freedom	Sum of squares
Regression over all weeks = S_1	$N-P'$*	$\sum_{i=1}^{128} y_{i.}^2(1-R_{.}^2)$
Regression for week 2	n_2-p	$\sum_{i=1}^{32} y_{i2}^2(1-R_2^2)$
Regression for week 3	n_3-p	$\sum_{i=1}^{32} y_{i3}^2(1-R_3^2)$
Regression for week 4	n_4-p	$\sum_{i=1}^{32} y_{i4}^2(1-R_4^2)$
Regression for week 5	n_5-p	$\sum_{i=1}^{32} y_{i5}^2(1-R_5^2)$
Sum of squared residuals for weeks 2-5 = S_2	$N-P$	$\sum_{i=1}^{32} \sum_{j=2}^{5} y_{ij}^2(1-R_j^2)$
Sum of squared residuals for weekly analysis = S_3	$(N-p) - (N-P'$*$)$	$[\sum_{i=1}^{128} y_{i.}^2(1-R_{.}^2) - \sum_{i=1}^{32} \sum_{j=2}^{5} y_{ij}^2(1-R_j^2))]$

*P' is distinguished from p in that P' represents the number of coefficients fit in the over-all equation while p is the number of coefficients estimated in each weekly equation.

Table 25.--Corrected sums of squares, multiple correlation coefficients and degrees of freedom by brand and week used in calculating the test statistic for price-carryover analysis for analytical Model I in estimating the demand relationships for frozen orange concentrate, 1964

Brand classification	Regression	Σy^2	R	Degrees of freedom
Nationally advertised	All weeks	69.08275	.68354	128-9 = 119
	Week 2	8.63646	.66194	32-6 = 26
	Week 3	12.79206	.68069	32-6 = 26
	Week 4	14.10883	.74093	32-6 = 26
	Week 5	17.20751	.74888	32-6 = 26
Packer	All weeks	124.39134	.85150	128-9 = 119
	Week 2	29.13435	.86260	32-6 = 26
	Week 3	28.66984	.81208	32-6 = 26
	Week 4	35.35235	.84529	32-6 = 26
	Week 5	26.13928	.93276	32-6 = 26
Private	All weeks	52.11162	.77599	128-9 = 119
	Week 2	13.80190	.83990	32-6 = 26
	Week 3	11.67943	.74777	32-6 = 26
	Week 4	9.51291	.80684	32-6 = 26
	Week 5	11.99647	.79828	32-6 = 26

Packer brand

$$\frac{\frac{3959.46126}{33}}{\frac{61065.91359}{80}} = .15718 \text{ with d.f.}\,\Big|^{33}_{80}$$

Private brand

$$\frac{\frac{25368.43482}{33}}{\frac{171384.65682}{80}} = .35883 \text{ with d.f.}\,\Big|^{33}_{80}$$

The price-carryover tests for analytical Model III, using the values shown in Table 27, were:

Nationally advertised brand

$$\frac{\frac{1403.77255}{33}}{\frac{15765.55801}{80}} = .21585 \text{ with d.f.}\,\Big|^{33}_{80}$$

Table 26.--Corrected sums of squares, multiple correlation coefficients and degrees of freedom by brand and week used in calculating the test statistic for price-carryover analysis for analytical Model II in estimating the demand relationships for frozen orange concentrate, 1964 .

Brand classification	Regression	Σy^2	R	Degrees of freedom
Nationally advertised	All weeks	54602.01465	.73691	128-15 = 113
	Week 2	14160.22315	.77495	32-12 = 20
	Week 3	15594.70874	.72503	32-12 = 20
	Week 4	13684.96265	.76090	32-12 = 20
	Week 5	10411.22131	.74619	32-12 = 20
Packer	All weeks	214110.55273	.83440	128-15 = 113
	Week 2	52581.06494	.86450	32-12 = 20
	Week 3	48151.39941	.83475	32-12 = 20
	Week 4	59131.86768	.83533	32-12 = 20
	Week 5	53849.00586	.84595	32-12 = 20
Private	All weeks	380199.21094	.69461	128-15 = 113
	Week 2	127708.01172	.76584	32-12 = 20
	Week 3	95078.50195	.75072	32-12 = 20
	Week 4	54117.57031	.75045	32-12 = 20
	Week 5	98359.37109	.67575	32-12 = 20

Packer brand

$$\frac{\frac{4429.38967}{33}}{\frac{31362.14503}{80}} = .34238 \text{ with d.f.}\Big|^{33}_{80}$$

Private brand

$$\frac{\frac{16565.64125}{33}}{\frac{63817.37041}{80}} = .62928 \text{ with d.f.}\Big|^{33}_{80}$$

The approximate $F_{.95}$ values with the respective degrees of freedom were:[5]

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[5] Mathematical Tables from Handbook of Chemistry and Physics, ed., C. D. Hodgman, S. M. Selby, and R. C. Weast (11th ed.; Cleveland, Ohio, Chemical Rubber Publishing Co., 1961), p. 222.

Table 27.--Corrected sums of squares, multiple correlation coefficients and degrees of freedom by brand and week used in calculating the test statistic for price-carryover analysis for analytical Model III in estimating the demand relationships for frozen orange concentrate, 1964

Brand classification	Regression	Σy^2	R	Degrees of freedom
Nationally advertised	All weeks	27312.29907	.60940	128-15 = 113
	Week 2	6391.10791	.77223	32-12 = 20
	Week 3	9029.11023	.58623	32-12 = 20
	Week 4	7971.35382	.62466	32-12 = 20
	Week 5	3828.33026	.61086	32-12 = 20
Packer	All weeks	130435.62207	.85184	128-15 = 113
	Week 2	30643.37524	.90509	32-12 = 20
	Week 3	26896.01440	.83188	32-12 = 20
	Week 4	36583.54346	.84798	32-12 = 20
	Week 5	34505.33057	.88856	32-12 = 20
Private	All weeks	220469.03906	.79712	128-15 = 113
	Week 2	53540.80371	.84557	32-12 = 20
	Week 3	43736.70313	.80955	32-12 = 20
	Week 4	44842.53662	.79255	32-12 = 20
	Week 5	74121.51172	.87932	32-12 = 20

$$F_{.95}\Big|^{15}_{104} \doteq 2.12$$

$$F_{.95}\Big|^{33}_{80} \doteq 1.69$$

All of the test statistic values except the nationally advertised brand in analytical Model I failed to fall in the critical region at the specified level of confidence. Thus, all but one of the hypotheses of one regression over all weeks could not be rejected by this test. The one rejected hypothesis suggested that further analysis of the regression coefficients, fit to the nationally advertised brand in analytical Model I, had to be carried out.

Testing for significant regression coefficients

Results of the price-carryover analysis limited the examination of the regression equations in each analytical model. For all brands in Models II and III, one regression for each brand over all weeks was sufficient for estimating price effects on sales. Within analytical Model I, one equation over all weeks for the packer and private labels respectively was analyzed, while the decision concerning one regression for the nationally advertised brand was withheld pending further investigation of the individual coefficients fit within weeks. It was assumed that the estimated value of each regression coefficient was drawn from a population of estimates of the same parameter which was normally and independently distributed.

In that a normal deviate divided by the square root of an unbiased estimate of its variance is distributed as "t" with the respective degrees of freedom,[6] the test of each individual regression coefficient was as follows:

$$H_2 : \hat{b}_{iK} - B_{iK} = 0$$
$$\alpha = .30$$

Test statistic:

$$t = \frac{b_{iK} - B_{iK}}{s_{biK}} \qquad (5.14)$$

Critical region:

$$|t| > t_{1-\frac{\alpha}{2}}, \text{ d.f.}$$

Failure to reject the null hypothesis for any coefficient at the test level implied only that these data did not include any measurable importance for the independent variable in question. Only further examination could resolve the problem of these nonsignificant coefficients. The same reasoning, which did not allow acceptance of the null hypothesis but rather specified reserved judgment pending further investigation of the coefficient involved, also led to the stipulation that the probability of having a calculated "t" value larger than $t_{1-\frac{\alpha}{2}}$, d.f. due to chance, would be P = .30. It was arbitrarily established that a 30 percent chance of rejecting a true hypothesis was allowable in that specification of the independent variables was done in a manner that included only those factors considered to be important.

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[6] R. L. Anderson and T. A. Bancroft, Statistical Theory in Research (New York: McGraw-Hill Book Co., Inc., 1952), p. 179.

The test statistic (5.14) resolved into the simple ratio of each regression coefficient to its respective standard error. The significant coefficients under the prescribed critical region are shown for all equations in the complete listing of coefficients and standard errors in Tables 20, 21 and 22. Derivation of demand elasticities and their interpretation utilized only the equations specified in the price-carryover tests and the significant coefficients within these particular regressions.

Interpretation of the Regression Equations

Regression coefficients calculated in the three analytical models represented estimates of the respective slopes of the demand surfaces for each brand--they did not measure the price elasticity of demand. These economic parameters were computed from the definition of elasticity of demand:

$$\frac{\partial Q}{\partial P} \cdot \frac{P}{Q} = E_p \qquad (5.15)$$

Derivation of elasticity coefficients

Analytical Model I.--While the hypothesis of one regression equation for all weeks for the nationally advertised brand was rejected in analytical Model I, the possibility still existed that the regression coefficients associated with the respective brand prices were estimates of the same population parameter. It was assumed that the coefficients in each equation were independent; therefore, the pooled variance for a test of no difference between any pair of constants drawn from the different equations was a simple sum. Moreover, the test statistic was distributed as "t" with $N_1 + N_2 - 2$ degrees of freedom. Each significant coefficient in the over-all regression equation was tested against the corresponding coefficient in the respective week equations. The null hypothesis and test criteria were established as:

$$H_3 : b_{iT} - b_{ik} = 0$$
$$\alpha = .05$$

Test statistic:

$$t = \frac{b_{iT} - b_{ik}}{\sqrt{s_{biT}^2 + s_{bik}^2}} \tag{5.16}$$

Critical region:

$$|t| > t_{1-\frac{\alpha}{2}}$$

where:

b_{iT} = the i^{th} coefficient in the over-all regression equation

b_{ik} = the i^{th} coefficient in the k^{th} week regression equation

Values of the respective test statistics were:

(1) $b_{1T} - b_{12} = 0$

$$\frac{-2.28077 - (-2.22115)}{\sqrt{(.53222)^2 + (.31229)^2}} = \frac{-.05962}{\sqrt{.380783}} = \frac{-.05962}{.61707} = -.09661$$

(2) $b_{1T} - b_{13} = 0$

$$\frac{-2.28077 - (-1.97451)}{\sqrt{(.31229)^2 + (.64089)^2}} = \frac{-.30626}{\sqrt{.50827}} = \frac{-.30626}{.71293} = -.42957$$

(3) $b_{1T} - b_{14} = 0$

$$\frac{-2.28077 - (-2.71399)}{\sqrt{(.31229)^2 + (.61834)^2}} = \frac{.43322}{\sqrt{.47987}} = \frac{.43322}{.69273} = .62538$$

(4) $b_{1T} - b_{15} = 0$

$$\frac{-2.28077 - (-2.16054)}{\sqrt{(.31229)^2 + (.67709)^2}} = \frac{-.12023}{\sqrt{.55598}} = \frac{-.12023}{.74564} = -.16124$$

(5) $b_{3T} - b_{32} = 0$

$$\frac{.28769 - .35156}{\sqrt{(.26454)^2 + (.46111)^2}} = \frac{-.06387}{\sqrt{.28260}} = \frac{-.06387}{.53160} = -.12014$$

(6) $b_{3T} - b_{33} = 0$

$$\frac{.28769 + .72786}{\sqrt{(.26454)^2 + (.53907)^2}} = \frac{1.01555}{\sqrt{.36058}} = \frac{1.01555}{.60048} = 1.69123$$

(7) $b_{3T} - b_{34} = 0$

$$\frac{.28769 - .57918}{\sqrt{(.26454)^2 + (.52272)^2}} = \frac{-.29149}{\sqrt{.34322}} = \frac{-.29149}{.58585} = -.49755$$

(8) $b_{3T} - b_{35} = 0$

$$\frac{.28769 - .97949}{\sqrt{(.26454)^2 + (.56533)^2}} = \frac{-.69180}{\sqrt{.38958}} = \frac{-.69180}{.62416} = -1.10836$$

d.f. in all tests $=(N_1 + N_2 - 2)=(32 + 128 - 2)= 158$

In all tests, the calculated "t" value did not fall in the critical region, indicating insufficient evidence to reject the hypothesis concerning no difference between the over-all price regression coefficients and the weekly price regression coefficients. Therefore, the over-all regression equation was regarded as sufficient for derivation of elasticity coefficients.

While analytical Model I was fit in natural logarithms, the general proof of the elasticity values was accomplished in arithmetic form. Equation (5.6) in arithmetic structure for brand 1 was defined as:

$$\frac{Y^*_1}{Y_1} = b_0 \left(\frac{X^*_1}{X_1}\right)^{b_1} \left(\frac{X^*_2}{X_2}\right)^{b_2} \left(\frac{X^*_3}{X_3}\right)^{b_3} \qquad (5.17)$$

Then, using (5.15) the price elasticity of demand for brand 1 was found by rewriting (5.17) in terms of Y_1 as:

$$Y_1 = \frac{Y^*_1}{b_0 \left(\frac{X^*_1}{X_1}\right)^{b_1} \left(\frac{X^*_2}{X_2}\right)^{b_2} \left(\frac{X^*_3}{X_3}\right)^{b_3}} \qquad (5.18)$$

or:

$$Y_1 = b_0 \left[\left(\frac{X^*_1}{1}\right)^{b_1} \left(\frac{X^*_2}{X_2}\right)^{b_2} \left(\frac{X^*_3}{X_3}\right)^{b_3}\right]^{-1} \cdot Y^*_1 X_1^{b_1} \qquad (5.19)$$

and differentiating with respect to X_1:

$$\frac{\partial Y_1}{\partial X_1} = b_1 \frac{Y^*_1 X_1^{b_1 - 1}}{b_0 \left(\frac{X^*_1}{1}\right)^{b_1} \left(\frac{X^*_2}{X_2}\right)^{b_2} \left(\frac{X^*_3}{X_3}\right)^{b_3}} \qquad (5.20)$$

Using (5.15) to calculate the price elasticity of demand for brand 1 gave:

$$\frac{\partial Y_1}{\partial X_1} \cdot \frac{X_1}{Y_1} = b_1 \frac{Y^*_1 X_1^{b_1 - 1}}{b_0 \left(\frac{X^*_1}{1}\right)^{b_1} \left(\frac{X^*_2}{X_2}\right)^{b_2} \left(\frac{X^*_3}{X_3}\right)^{b_3}} \cdot \frac{X_1}{Y_1} \qquad (5.21)$$

which resulted in:

$$b_1 \frac{{Y^*}_1 X_1^{b_1 - 1 + 1}}{b_0 \left(\frac{{X^*}_1}{1}\right)^{b_1} \left(\frac{{X^*}_2}{X_2}\right)^{b_2} \left(\frac{{X^*}_3}{X_3}\right)^{b_3}} \cdot \frac{b_0 \left(\frac{{X^*}_1}{1}\right)^{b_1} \left(\frac{{X^*}_2}{2}\right)^{b_2} \left(\frac{{X^*}_3}{X_3}\right)^{b_3}}{{Y^*}_1 X_1^{b_1}} \qquad (5.22)$$

therefore:

$$E_{D1.1} = \frac{\partial Y_1}{\partial X_1} \cdot \frac{X_1}{Y_1} = b_1 \qquad (5.23)$$

Derivation of the cross-price elasticity coefficients for brand 1 by the same method resulted in:

$$E_{D1.2} = \frac{\partial Y_1}{\partial X_2} \cdot \frac{X_2}{Y_1} = b_2 \qquad (5.24)$$

$$E_{D1.3} = \frac{\partial Y_1}{\partial X_3} \cdot \frac{X_3}{Y_1} = b_3 \qquad (5.25)$$

Analytical Model II.--The matrix of independent price variables in analytical Model II was obtained by subtracting the mean prices from each respective price level and multiplying the result by a constant term. The subtraction of a constant term from the independent variables did not alter the slope estimates in each equation. However, in order to obtain the values of the regression coefficients and their standard errors in the original price scales, each coefficient and error term was multiplied by the constant term, .2396. Significance tests in either scale gave the same results, so elasticity estimates were derived in the original price variables. Thus, equation (5.7), after transformation back to the original price levels by multiplying the respective variables by .2396, was written for brand 1 as:

$$\begin{aligned} Y_1 &= b_0 + (X_1-X^*_1) + b_2(X_2-X^*_2) + b_3(X_3-X^*_3) \\ &\quad + b_4(X_1-X^*_1)^2 + b_5(X_2-X^*_2)^2 + b_6(X_3-X^*_3)^2 \\ &\quad + b_7(X_1-X^*_1)(X_2-X^*_2) + b_8(X_1-X^*_1)(X_3-X^*_3) \\ &\quad + b_9(X_2-X^*_2)(X_3-X^*_3) \end{aligned} \tag{5.26}$$

Expanding the right-hand side of (5.26) gave:

$$\begin{aligned} Y_1 &= b_0 + b_1X_1 - b_1X^*_1 + b_2X_2 - b_2X^*_2 + b_3X_3 \\ &\quad - b_3X^*_3 + b_4X_1^2 - 2b_4X_1X^*_1 + b_4X^{*2}_1 \\ &\quad + b_5X_2^2 - 2b_5X_2X^*_2 + b_5X^{*2}_2 + b_6X_3^2 \\ &\quad - 2b_6X_3X^*_3 + b_6X^{*2}_3 + b_7X_1X_2 - b_7X^*_1X_2 \\ &\quad - b_7X_1X^*_2 + b_7X^*_1X^*_2 + b_8X_1X_3 - b_8X^*_1X_3 \\ &\quad - b_8X_1X^*_3 + b_8X^*_1X^*_3 + b_9X_2X_3 - b_9X^*_2X_3 \\ &\quad - b_9X_2X^*_3 + b_9X^*_2X^*_3 \end{aligned} \tag{5.27}$$

Differentiating this expansion with respect to X_1 gave:

$$\begin{aligned} \frac{\partial Y_1}{\partial X_1} &= b_1 + 2b_4X_1 - 2b_4X^*_1 + b_7X_2 - b_7X^*_2 \\ &\quad + b_8X_3 - b_8X^*_3 \end{aligned} \tag{5.28}$$

At the mean price level, $X_1 = X^*_1$, $X_2 = X^*_2$, and $X_3 = X^*_3$; therefore, the price elasticity of demand for brand 1 at the mean price level was calculated as:

$$E_{D1.1} = \frac{\partial Y_1}{\partial X_1} \cdot \frac{X_1}{Y_1} = b_1 \cdot \frac{X_1}{Y_1} \tag{5.29}$$

Similarly, the cross-price elasticities of demand for brand 1 were:

$$E_{D1.2} = \frac{\partial Y_1}{\partial X_2} \cdot \frac{X_2}{Y_1} = b_2 \cdot \frac{X_2}{Y_1} \tag{5.30}$$

$$E_{D1.3} = \frac{\partial Y_1}{\partial X_3} \cdot \frac{X_3}{Y_1} = b_3 \cdot \frac{X_3}{Y_1} \tag{5.31}$$

Analytical Model III.--The regression equations fit in Model III were the arithmetic counterparts of those equations fit using logarithmic differences in Model I. Rewriting the ratios in (5.6) in arithmetic differences and adding square and cross-product terms gave for brand 1:

$$\begin{aligned} Y^{*}_{1} - Y_{1} = {} & b_{0} + b_{1}(X^{*}_{1}-X_{1}) + b_{2}(X^{*}_{2}-X_{2}) + b_{3}(X^{*}_{3}-X_{3}) \\ & + b_{4}(X^{*}_{1}-X_{1})^{2} + b_{5}(X^{*}_{2}-X_{2})^{2} + b_{6}(X^{*}_{3}-X_{3})^{2} \\ & + b_{7}(X^{*}_{1}-X_{1})(X^{*}_{2}-X_{2}) + b_{8}(X^{*}_{1}-X_{1})(X^{*}_{3}-X_{3}) \\ & + b_{9}(X^{*}_{2}-X_{2})(X^{*}_{3}-X_{3}) \end{aligned} \qquad (5.32)$$

Rewriting (5.32) in terms of Y_1 gave:

$$\begin{aligned} Y_{1} = {} & Y^{*}_{1} - b_{0} - b_{1}X^{*}_{1} + b_{1}X_{1} - b_{2}X^{*}_{2} + b_{2}X_{2} \\ & - b_{3}X^{*}_{3} + b_{3}X_{3} - b_{4}X^{*2}_{1} + 2b_{4}X_{1}X^{*}_{1} - b_{4}X_{1}^{2} \\ & - b_{5}X^{*2}_{2} + 2b_{5}X_{2}X^{*}_{2} - b_{5}X_{2}^{2} - b_{6}X^{*2}_{3} \\ & + 2b_{6}X_{3}X^{*}_{3} - b_{6}X_{3}^{2} - b_{7}X^{*}_{1}X^{*}_{2} + b_{7}X^{*}_{1}X_{2} \\ & + b_{7}X_{1}X^{*}_{2} - b_{7}X_{1}X_{2} - b_{8}X^{*}_{1}X^{*}_{3} + b_{8}X^{*}_{1}X_{3} \\ & + b_{8}X_{1}X^{*}_{3} - b_{8}X_{1}X_{3} - b_{9}X^{*}_{2}X^{*}_{3} + b_{9}X^{*}_{2}X_{3} \\ & + b_{9}X_{2}X^{*}_{3} - b_{9}X_{2}X_{3} \end{aligned} \qquad (5.33)$$

Differentiating (5.33) with respect to X_1 resulted in:

$$\begin{aligned} \frac{\partial Y_{1}}{\partial X_{1}} = {} & b_{1} + 2b_{4}X^{*}_{1} - 2b_{4}X_{1} + b_{7}X^{*}_{2} - b_{7}X_{2} \\ & + b_{8}X^{*}_{3} - b_{8}X_{3} \end{aligned} \qquad (5.34)$$

Derivation of the price elasticity of demand for brand 1 at the mean price level where $X_1 = X^{*}_{1}$, $X_2 = X^{*}_{2}$, $X_3 = X^{*}_{3}$ gave:

$$E_{D1.1} = \frac{\partial Y_{1}}{\partial X_{1}} \cdot \frac{X_{1}}{Y_{1}} = b_{1} \cdot \frac{X_{1}}{Y_{1}} \qquad (5.35)$$

Similarly, for the cross-price elasticities for brand 1:

$$E_{D1.2} = \frac{\partial Y_{1}}{\partial X_{2}} \cdot \frac{X_{2}}{Y_{1}} = b_{2} \cdot \frac{X_{2}}{Y_{1}} \qquad (5.36)$$

$$E_{D1.3} = \frac{\partial Y_{1}}{\partial X_{3}} \cdot \frac{X_{3}}{Y_{1}} = b_{3} \cdot \frac{X_{3}}{Y_{1}} \qquad (5.37)$$

Evaluation of the Alternative Analytical Models

The price-elasticity and cross-elasticity coefficients derived for each respective model are shown in Table 28.

Table 28.--Price elasticities and cross-price elasticities of demand calculated over all weeks and brands using analytical Models I, II and III in estimating the demand relationships for frozen orange concentrate, 1964

Brand sales	Price		
	Nationally advertised	Packer	Private
	Model I elasticity coefficients		
Nationally advertised	-2.28077	--	+0.28769
Packer	+0.80669	-4.37062	+1.17141
Private	+0.92884	+0.55157	-2.56857
	Model II elasticity coefficients		
Nationally advertised	-2.61129	+0.40859	+1.00129
Packer	--	-2.99119	+1.50801
Private	-0.46174	-0.35697	-1.83398
	Model III elasticity coefficients		
Nationally advertised	-2.46187	--	--
Packer	--	-2.93369	+0.73087
Private	+0.28174	+0.27184	-2.06451

The three analytical procedures utilized in approximating the economic model differ in the makeup of the variables within the equation formats. The basic input data represented a series of observations from what was assumed to be an ordered socioeconomic system reacting to a series of artificially induced price situations. The character of the response measured in the sales data, therefore, cannot be changed by algebraic alterations of the estimating regression functions. Rather, these mathematical manipulations comprise a methodological probe designed to approximate the observed order in the best possible manner. The criterion for specifying which estimates were best for evaluating the postulated economic model, however, was divorced from purely statistical techniques, and was more strongly influenced by economic theory and the practical aspects of interpretation.

Economic consistency

Conformance of the various price elasticities to certain economic principles was of primary concern in the investigation of the various estimates derived in the preceding chapter. The basis for these requirements was established earlier in the acceptance of the phenomenon of diminishing marginal utility adhered to in the development of the economic model. Derivation of a demand curve using a basic marginal utility curve for one good depicted this function as sloping downward and to the right; or for one brand the effect of this brand's price on its sales was assumed to be an inverse relationship. Thus, for the price elasticities of demand, the following consistency requirements were made:

$$E_{D1.1} < 0 \tag{5.38}$$

$$E_{D2.2} < 0 \tag{5.39}$$

$$E_{D3.3} < 0 \tag{5.40}$$

An additional specification was made concerning the effect of one brand's price on the sales of the competing brands relative to its own sales. The price of an individual brand was assumed to have the greatest influence on sales of that brand. The total substitution effects of the remaining two brands could not exceed this direct price elasticity. Formally stated, this requirement was:

$$|E_{D1.1}| > |E_{D2.1} + E_{D3.1}| \quad (5.41)$$

$$|E_{D2.2}| > |E_{D1.2} + E_{D3.2}| \quad (5.42)$$

$$|E_{D3.3}| > |E_{D1.3} + E_{D2.3}| \quad (5.43)$$

The three brands of frozen orange concentrate were in the same class of generic goods differentiated in the consumer's mind on a label and price basis. These items were, therefore, considered as a category of competing products. This specification led to the consistency requirement that the substitution effects had to be positive and--a corollary of the preceding requirement--could not exceed the direct price elasticity for each brand.

$$\begin{aligned} &E_{D1.2} > 0,\ E_{D1.3} > 0 \\ &E_{D1.1} + E_{D1.2} + E_{D1.3} < 0 \end{aligned} \quad (5.44)$$

$$\begin{aligned} &E_{D2.1} > 0,\ E_{D2.3} > 0 \\ &E_{D2.2} + E_{D2.1} + E_{D2.3} < 0 \end{aligned} \quad (5.45)$$

$$\begin{aligned} &E_{D3.1} > 0,\ E_{D3.2} > 0 \\ &E_{D3.3} + E_{D3.1} + E_{D3.2} < 0 \end{aligned} \quad (5.46)$$

The first consistency test for all three analytical models gave the following results:

Model I

$E_{D1.1} = -2.28077$

$E_{D2.2} = -4.37062$

$E_{D3.3} = -2.56857$

Model II

$E_{D1.1} = -2.61129$

$E_{D2.2} = -2.99119$

$E_{D3.3} = -1.83398$

Model III

$E_{D1.1} = -2.46187$

$E_{D2.2} = -2.93369$

$E_{D3.3} = -2.06451$

All three analytical models provided elasticity estimates that satisfied the first requirement.

The second series of economic consistency tests resulted in the following calculations:

Model I

$|-2.28077| > |.80689 + .928891|$

$|-2.28077| > |1.73553|$

$|-4.37062| > |0 + .55157|$

$|-4.37062| > |.55157|$

$|-2.56857| > |.28769 + 1.17141|$

$|-2.56857| > |1.45910|$

Model II

$|-2.61129| > |0 - .46174|$

$|-2.61129| > |-.46174|$

$|-2.99119| > |.40859 - .35697|$

$|-2.99119| > |.05162|$

$|-1.83398| > |1.00129 + 1.50801|$

$|-1.83398| \not> |2.50930|$

Model III

$|-2.46187| > |0 + .28174|$

$|-2.46187| > |.28174|$

$|-2.93369| > |0 + .27184|$

$|-2.93369| > |.27184|$

$|-2.06451| > |0 + .73087|$

$|-2.06451| > |.73087|$

While analytical Models I and III satisfied this requirement, Model II did not. Model II was discarded at this point because the signs of $E_{D3.1}$ and $E_{D3.2}$ were negative, indicating that a rejection by the subsequent consistency test would also be forthcoming. The underlying cause of these inconsistencies was attributed to the rewriting of the difference equation format for this model. The basis for rewriting the format was the assumption that all store variability not handled by traffic adjustments would be accounted for in the quantity intercept estimate. Examination of the sales inputs for this analysis revealed that an inverse relationships actually did exist between the prices of brands 1 and 2 and the sales of brand 3 across all stores. Subsequent investigation of the sales difference versus price changes as employed in Models I and III indicated that the relative quantity changes over this same brand-price situation were positive, strengthening the concept of difference equations to compensate for store variability.

The final consistency test was made for Models I and III.

Model I

$-2.28077 + 0 + .28769 = -1.99308 < 0$

$-4.37062 + .80669 + 1.17141 = -2.39252 < 0$

$-2.56857 + .92884 + .55157 = -1.08816 < 0$

Model III

$$-2.46187 + 0 + 0 = -2.46187 < 0$$

$$-2.93369 + 0 + .73087 = -2.20282 < 0$$

$$-2.06451 + .28174 + .27184 = -1.51093 < 0$$

The various elasticities in Model I and Model III satisfied the last consistency requirement. Selection of the set of parameters best suited for use in the evaluation phase resolved into a further investigation of the nature of logarithmic versus arithmetic demand curves.

Logarithmic versus arithmetic demand curves

There are several statistical considerations under which logarithmic transformations are desirable prior to fitting functions to economic data. The most common of these pertains to a priori reasoning indicating multiplicative relationships in the model format. A hypothetical equation representing this property could be depicted as:

$$Y = aX_1^{b_1}X_2^{b_2} \qquad (5.47)$$

where the relationship between the dependent variable Y and the two independent variables X_1, X_2 is a product combination in the exponents b_1 and b_2. If (5.47) is rewritten in logarithms, then:

$$\ln Y = \ln a + b_1 \ln X_1 + b_2 \ln X_2 \qquad (5.48)$$

and the resulting function (5.48) is linear in logarithms with the effects of the two independent variables considered additive in this dimension.

Two other cases in which a logarithmic transformation might be considered desirable are also concerned with the problem of proportionality. If the researcher has reason to suspect that a more stable relationship exists among certain variables when the data is expressed in percentage rather than absolute terms, then a logarithmic function should be considered. Secondly, if the unexplained residuals are suspected to be more uniform under the same conditions, a logarithmic transformation would be desirable.[7] However, these considerations are designed to strengthen the objectivity of statistical analysis of economic data from the standpoint of the assumption of additivity supporting experimental design and regression techniques. The question of interpretation of the results of logarithmic equations in economic terms is not resolved by these considerations. Specifically, the usefulness of elasticity estimates derived through logarithmic demand curves as in this study, while partially related to proper statistical techniques, is not solely governed by mathematical considerations.

The main interpretive advantage in logarithmic demand curves is that the various price elasticities are calculated directly in the regression procedure. The regression constants are estimates of an average elasticity over the range of the price variable used. This advantage, however, is sometimes cause for concern from the standpoint of economic theory. This concern stems from the fact that a logarithmic demand curve has constant elasticity throughout the price range involved. If this range is relatively wide compared to all possible prices that could be charged for the commodity in question, then the economic consistency of such a demand curve breaks down. An alternative type of function fit using nontransformed data will give results which at first approximation have more economic usefulness than the constant elasticities derived from logarithmic functions.

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[7] R. J. Foote, Analytical Tools for Studying Demand and Price Structures, U. S. Dept. of Agriculture Handbook No. 146 (Washington: U. S. Government Printing Office, 1958), p. 37.

The reason that price elasticities derived from arithmetic regression equations follow a pattern consistent with economic theory can be seen in the derivation of the price elasticity of demand for a single commodity. If the regression line approximating this relationship is given as:

$$Y = a + bX \qquad (5.49)$$

where:

Y = the quantity sold of a product

a = the quantity intercept for $X = 0$

X = the price of the product Y

b = the regression estimate of the slope of the relationship between Y and X

then, assuming the function to be continuous, the price elasticity of demand for Y is given as:

$$E_D = \frac{dY}{dX} \cdot \frac{X}{Y} \qquad (5.50)$$

Calculating E_D for equation (5.49) gives:

$$E_D = \frac{dY}{dX} \cdot \frac{X}{Y} = b \cdot \frac{X}{Y}$$

and:

$$b \cdot \frac{X}{Y} = b \cdot \frac{X}{a + bX}$$

Thus, the price elasticity of demand is a combined effect of the slope of the regression line, the value of the independent variable and the corresponding predicted quantity at a given price level. In this manner, E_D will take on different values depending upon the level of price, thus representing a series of point elasticities along a given demand function.

In this study, analytical Model I represented a linear logarithmic function within a difference equation technique adopted specifically for store variability compensation. Model III utilized the same difference equation procedure but was not transformed to logarithms or restricted to linear components. Rather, the absolute differences of both dependent and independent variables were used in the model, with the independent variables expanded to include interactions and quadratic terms. The derived elasticity estimates for both functions were close to one another in terms of magnitude, but differed to the extent of comparing an average to a point estimate. Also, the logarithmic curve passed through the geometric mean of the independent variable ratios, while the elasticity estimates from the arithmetic curve were derived at the arithmetic mean of the differences in the independent variables. Thus, both functions provided estimates of the same parameters. The major difference between the techniques was that Model I represented a series of average elasticities over the entire test price range while Model III approximated the same series of estimates at one price level. The prospect of selecting one of these equation formats for economic analysis was centered around this basic difference between the elasticity estimates.

Comparative utilization of demand elasticities

Investigation of short-run changes in quantity demanded along the function given in Model III is not a simple process. Moreover, the difference equation construction in this regression model, while necessary from the standpoint of adjusting the data for store-to-store variation, does not allow prediction of the actual quantity sold at each price level. This same limitation applies equally to the logarithmic function in Model I. However, the seriousness of this problem for Model III is that derivation of elasticities of demand using (5.15) requires a predicted quantity value. The dependent variable in Model III is the absolute change in quantity sold; therefore, derivation of the various elasticity estimates at any price other than the average price level cannot be accomplished by direct differentiation procedures. The only means whereby price elasticity and cross-price elasticity coefficients can be determined from Model III is to first calculate the actual quantity changes for each price change, with the nonprice independent variables held at their respective average levels. The next step is to find the new total quantity sold by adding or subtracting the respective change from the mean sales figure. Finally, the new total quantity and formula (5.50) must be used for defining a new point elasticity estimate. The calculation of point elasticity values under this process is a trivial matter in that the change in quantity sold for each price shift is computed directly from the regression equations. Even if the point elasticities were calculated, their usefulness would be limited in any interpretative process because infinitesimally small price changes were not made. In such a case, a simple arc elasticity formula using the direct price variables and the estimated quantity changes would be more meaningful.

The logarithmic function fit in analytical Model I, while conceptually utilizing a predicted quantity for derivation of the resective demand elasticities, does not require price or quantity levels in the final interpretation. Thus, the elasticity coefficients, which represent averages over the range of prices used in this study, can be used to estimate quantity changes directly. In that arc elasticities arrived at by a rather roundabout procedure using Model III also represent averages between given price levels, it was subsequently decided to use the direct results of analaytical Model I in the economic analysis.

CHAPTER VI

IMPLICATIONS OF THE RESULTS

The general evaluation of the economic model which follows in this chapter is set within the broad spectrum of what has frequently been called the marketing concept.[1] The marketing concept does not include a series of guidelines, compilation of assumptions, and theories of operation similar to the clear-cut marginal analysis applied in economic principles. Rather, this philosophy regards any firm as a consumer-directed business complex. The ultimate objective of a company remains unchanged; that is, that the best possible profit position be maintained over the life of the enterprise. However, the translation of this broad policy into operating goals and the means of obtaining these goals takes on the form of a refined process of analyzing and adjusting to the marketplace. The deliberation of management thus becomes directed towards the formulation of a over-all market strategy.

A marketing strategy consists of a definition of a marketing mix in terms of identification of types of customers whose patronage is to be sought, and market areas and new product potentials to be expanded.[2] The specification of these objectives, or market targets, involves an up-to-date knowledge of consumers' wants and desires with reference to the product a firm sells. In addition, the competitive position held by a firm in a particular market is of importance in formulating the objectives within a marketing strategy. The marketing mix is comprised of a blend of means whereby the operating objectives are to be achieved. There are four basic classifications of factors which must be considered in establishing a desired marketing mix.[3] These classifications are:

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[1] J. B. McKitterick, "What is the Marketing Management Concept?" Managerial Marketing: Perspectives and Viewpoints, ed. W. Lazer and E. J. Kelley (rev. ed.; Homewood, Ill.: Richard D. Irwin, Inc., 1962).

[2] A. R. Oxenfeldt, "The Formulation of a Market Strategy," Managerial Marketing: Perspectives and Viewpoints, ed. W. Lazer and E. J. Kelley (rev. ed.; Homewood, Ill.: Richard D. Irwin, Inc., 1962).

(1) The nature of the product being sold

(2) The physical location and manner in which the product is being sold

(3) The types of promotional devices being used in reference to the product

(4) The price that is being charged for the product

The optimum combination of means to achieve a desired goal, whether the goal be inducing customers to change to the product a firm sells or strengthening and expanding the market for a commodity of an entire industry without reference to any competition, can only result from an awareness of the effects of these four items. The product must not only appeal to consumers, but must be an entity that possesses utility-producing characteristics which definitely satisfy some desire of the person seeking to purchase it. The general atmosphere and merchandising structure of the retail market should enhance the appearance and quality of the commodity, as well as support any promotional and price competition which may be pertinent to a particular area. The effects of advertising programs at all sales levels cannot be ignored in structuring a promotional campaign integrated with product features, merchandising procedures and pricing policies. Finally, the price to be charged for a product must be commensurate with cost and revenue considerations, the consistency of the price to the customers' image of the product, and consumer awareness of and sensitivity to price.

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[3] Oxenfeldt, p. 40.

Customer awareness of and sensitivity to price is the specific marketing area with which this study is concerned. Interpretation of the elasticity coefficients using analytical Model I, along with the implications of the market reactions derived therefrom, will be within the framework of a short-run strategy complex for the orange industry.

Demand and Substitution Relationships

The elasticity coefficients calculated from analytical Model I are listed in Table 29. The ensuing discussion will be in terms of these coefficients and the respective price-quantity relationships they represent.

Table 29.--Price-elasticity and cross-elasticity coefficients calculated using analytical Model I in estimating the demand and substitution relationships for frozen orange concentrate, 1964

Brand sales	Elasticity coefficients		
	Nationally advertised price	Packer price	Private price
Nationally advertised	-2.28077	--	0.28769
Packer	0.80669	-4.37062	1.17141
Private	0.92884	0.55157	-2.56857

The total demand structure in terms of each isolated segment is approximated by the elasticity coefficients relating the sales of each brand to its respective price. Thus, a one percent change in the price of the nationally advertised brand will have an inverse effect on the national brand's sales of 2.28077 percent. The same situation generated for the packer and private brands will result in an inverse quantity change of 4.37062 and 2.56857 percent respectively. These price shifts are assumed to be accomplished in a system in which no substitution exists among the three brands. If this were the nature of the relationships for the data generated in this study, then all of the remaining elasticity coefficients in Table 29 would be zero. In such a system, the elastic character of the individual demand curves would be the most striking feature, with the total revenue curve for each brand category increasing with each price cut throughout the range of prices used in this study. The process of formulating a strategy relating to short-run price manipulation based on these coefficients alone, however, would be meaningless due to the existence of economic substitution among the brands.

The extent of substitution effects among the three brands of frozen orange concentrate is illustrated by the cross-elasticity coefficients relating price changes for one label to quantity changes in the other two labels. While the negative signs on the price elasticity coefficients indicate an inverse price-quantity relationship, the positive cross-elasticity values represent a direct movement in quantity sold compared to price shifts. Thus, a decrease in the private brand's price of one percent will cause a decrease in the national brand's sales by 0.28769 percent, while estimates indicate that fluctuations in the packer brand's price will have no effect on the sales of the national brand. The packer brand, however, faces competition from both the national and private label to the extent of 0.80669 and 1.17141 percent loss in sales as the respective prices of these two brands are reduced by one percent. The same situation exists for the private brand, with the national price affecting private sales by 0.92884 percent, and the packer price having the same influence in the amount of 0.55157 percent.

Marketing policy viewpoints

Interpreting the preceding elasticity components into a potential decision-making system in terms of consumer behavior patterns and marginal utility would be a rather complex procedure, ultimately being translated into quantity and price expectations for policy formulation. The results of the analytical procedure indicate the existence of relationships among the price and quantity variables consistent with economic theory and the psychology of consumer behavior. The usefulness of this information in terms of a long-range marketing structure for the citrus industry requires further evaluation. Before this can be done, however, the role of short-run market demand information in long-range planning should be clarified.

Long-range planning, rather than being a process of forecasting, is a result of the lack of ability to accurately forecast future events.[4] Uncertainty as to technological innovations, changes in market structure and the possibility of adverse weather conditions is a reality for all phases of orange producing and marketing firms. The major question facing policy formulation in this industry refers to what must be done in the present to prepare for an uncertain future situation. Elimination of future risk is not the operational goal; rather, current decisions are made concerning what risks are worth taking based on present information and expectations. Thus, long-range planning is a process of deciding what future risks must be taken under varying future conditions, and implementing these decisions in the short-run. The objectives to be attained, the assumptions concerning the nature of the market environment, the alternative courses of action that can be taken, and the expectation of results of the alternatives are major considerations in this process. The roles of product qualities, promotion, place or regional effects, and price are all interwoven into these considerations. The price effect under alternative decisions is the single factor explored in this study.

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[4] P. F. Drucker, "Long-Range Planning," Managerial Marketing: Perspectives and Viewpoints, ed. W. Lazer and E. J. Kelley (rev. ed.; Homewood, Ill.: Richard D. Irwin, Inc., 1962).

The demand and substitution relationships mentioned previously bear more significance in developing a marketing strategy when considered as a complete system rather than individual sets. The estimated existence of different price elasticity values and substitution effects implies that a very pluralistic market situation exists among the three brands. Within this structure, a single price change for a given brand as well as multiple price changes for all brands has a combined and interrelated effect on concentrate sales. Measurement of this effect requires calculation of a new component, free of a single dimension price variable, which represents the change in the individual quantities of each brand sold within the system. This component, called a net price effect, is calculated as shown in Table 30.

Table 30.--Calculation of net price effects for a constant percentage price change for all brands using elasticity coefficients from analytical Model I in estimating the demand relationships for frozen orange concentrate, 1964

Brand	Elasticity coefficients, $E_{D_{ij}}$			Net price effect, $\Sigma E_{D_{ij}}$
	Nationally advertised price P_1	Packer price P_2	Private price P_3	
1 Nationally advertised	-2.28077	--	0.28769	-1.99308
2 Packer	0.80669	-4.37062	1.17141	-2.39252
3 Private	0.92884	0.55157	-2.56857	-1.08816

This net effect represents the change in sales of each respective brand assuming that the percentage price changes for each label are the same. Conceptually, this component is the algebraic sum of the direct price effect and the cross-price effects for each brand equation. The expected results of the price manipulation under various objectives for the orange industry can be examined within the framework of these components.

Proportionate price changes.--One of the major considerations involving a price change for orange concentrate concerns the change in volume sold and the relative market share position of each brand class in this volume. Net price effects can be used to ascertain quantity and market share positions which can be expected under certain price changes on the retail market. The figures in Table 31 illustrate this process for the sales and price data in this study, assuming a constant 10 percent price change across all brands.

Table 31.--Changes in quantity sold by brand utilizing net price effects calculated from Model I elasticities in estimating the demand relationships for frozen orange concentrate, 1964

Brand	Average sales in ounces/300 customers	Market share percent	Net price effect	Price change percent	Quantity change results				
					Quantity change ounces/300 customers	Price change = +10.0 percent		Price change = -10.0 percent	
						New total quantity ounces/300 customers	Market share percent	New total quantity ounces/300 customers	Market share percent
Nationally advertised	23.22789	15.61	-1.99308	±10.0	∓ 4.62950	18.59839	14.90	27.85739	16.12
Packer	43.73739	29.39	-2.39252	±10.0	∓10.46426	33.27313	26.66	54.20165	31.36
Private	81.85464	55.00	-1.08816	±10.0	∓ 8.90709	72.94755	58.44	90.76173	52.52
Totals	148.81992	100.00	--	--	∓24.00085	124.81907	100.00	172.82077	100.00

In the event that the price of all three brands is raised by 10 percent, the packer label loses the greatest absolute sales volume, with the private and national brands losing by smaller amounts as a direct result of the differing net price effects and beginning average sales volume for each brand. At the new sales level, the private brand has actually gained a percentage share of the market at the expense of the packer and national labels. Conversely, a 10 percent decrease in the price of each brand will result in the same absolute gain ordering of the products, with the market share positions reversing. The packer and national brands will gain in market share position at the expense of the private brand. Figure 4 shows this trend in terms of percentage changes.

Thus, contemplation of price increases or decreases by all marketing firms handling concentrate should include the nature of the retail price change and the resulting quantity and market share alternatives. The prospect of increased revenue from a price decrease is suggested by the net price effects calculated for the regions in this study; however, if maintenance of internal relative market positions was desired by industry as a whole, a constant price shift by all firms would not be in order.

Disproportionate price changes.--If the operating goal of planning agencies within the orange industry was to maintain equal market shares for all firms in the concentrate market, even at the expense of added total revenue, then consideration of net price effects would be shifted from constant prices to constant percentage quantity changes. This procedure involves defining a desired quantity change and solving for the price alterations that will affect this change.

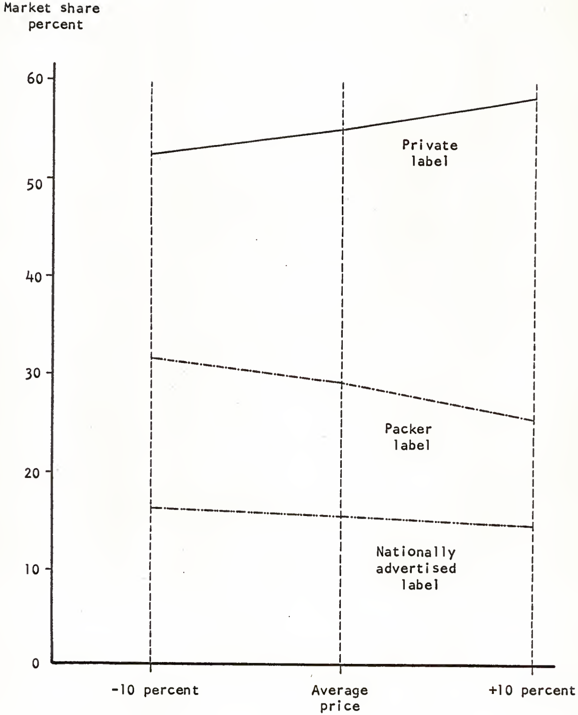

Figure 4.--Market share percentages by label for a plus or minus 10 percent price change

If a 10 percent increase in sales of all brands in the study region was desired, then using the data from Table 30, and solving:

$$-2.28077X_1 + 0X_2 + .28769X_3 = 10.0 \tag{6.1}$$

$$.80669X_1 - 4.37062X_2 + 1.17141X_3 = 10.0 \tag{6.2}$$

$$.92884X_1 + .55157X_2 - 2.56857X_3 = 10.0 \tag{6.3}$$

where the numerical values are the elasticity coefficients taken from Table 30 and the X_i terms are the respective percentage price changes, then the solutions:

$$X_1 = -5.253$$

$$X_2 = -5.099$$

$$X_3 = -6.888$$

represent the respective percentage price changes sought. In this example, if a 10 percent increase in sales of all three brands was desired with each label retaining the same market share, the retail price of the national brand would have to be reduced by 5.25 percent, the packer price by 5.10 percent, and the private price by 6.89 percent.

Summary of effects.--The ramifications of the various price effects in a pluralistic market as envisioned in this study are, in themselves, only expectations conditioned by uncertainty. The decision to follow a particular course of action with reference to retail pricing policy should not be based entirely on the preceding analysis but should also take into account regional products and time differences in relation to this study. Preparations for future conditions based on tendencies conceptualized in this system of elasticities can be executed with recognition that these relationships are only tendencies. Alteration of the operating goals specified in the earlier examples will not change the analytical systems or the process of finding solutions; rather, specification of different goals would result in new situations which would require further examination by marketing strategists prior to a final decision-making process.

The nature of the demand elasticities in single dimensions, while implying complete price-quantity definition, becomes clouded when the cross-elasticity coefficients are introduced. Utilization of net price effects and simultaneous equations, while seeming to complicate derivation of results under certain conditions, actually clarify procedures which must be followed given a desired operating goal. Finally, in consideration of short-run pricing policy, the possibility exists that immediate reactions might be varied for different brand segments when the market situation is reviewed over time. All of these factor effects are a combination of interactions which must be taken into account in the decision-making process of a complex of marketing firms. The final clarification of this segmented and yet interrelated demand structure is the derivation of the total demand elasticity for frozen orange concentrate.

The basic assumption underlying the estimation of the price elasticity of demand for all orange concentrate is that the total of all sales of the individual brands at their respective price levels defines a point on this commodity demand curve. Using data from Table 31, the commodity elasticity can be calculated at the mean study price level using the following arc-elasticity formula:

$$\frac{\dfrac{\Delta Q}{\dfrac{Q_1 + Q_2}{2}}}{\dfrac{\Delta P}{\dfrac{P_1 + P_2}{2}}} = E_D \qquad (6.4)$$

The total price elasticity of demand for the region used in this study for a 10 percent price decrease was calculated as:

$$E_D = \frac{\dfrac{+24.00085}{\dfrac{148.81992 + 172.82077}{2}}}{-.10} = \frac{.14923}{-.10} = -1.4923$$

Conversely, for a 10 percent price increase:

$$E_D = \frac{\dfrac{-24.00085}{\dfrac{148.81992 + 124.81907}{2}}}{+.10} = \frac{-.17542}{+.10} = -1.7542$$

Thus, the total price elasticity of demand at plus or minus 10 percent changes from the mean price level in this study was relatively elastic.

General Implications

Frozen orange concentrate, even when the degrees of economic substitution among the various brands within this commodity are considered, has an elastic demand curve within the price ranges specified in this study. This conclusion is to be expected given the large numbers of synthetic items existing in the juice market which are regarded as competitors for this product. From the standpoint of total revenue, price decreases will be accompanied by diminishing revenue expansions as the lower prices are reached. Such decreases, however, will necessarily be accompanied by shifts in purchasing patterns within the various brands of frozen orange concentrate to the extent that they compete with one another.

The formulation of future marketing strategies for the orange industry in Florida cannot be accomplished without information concerning the various phases of revenue and price considerations which are natural entities in demand analysis. The critical problem of selling large quantities of oranges through frozen orange concentrate outlets will be intensified as larger supplies of fresh fruit are made available in Florida. Under a demand structure similar to the one approximated in this study, a policy of high retail prices and withholding supply would be costly. The elastic character of the total demand curve and the net price effects realized in the segmented analysis suggest that price cuts commensurate with desired market share policy could lead to an ease in supply pressure as well as an increase in total revenue for the orange industry. An over-all strategy incorporating the various costs related to given output and movement levels would help to further clarify the situation from the standpoint of planning in the short-run.

The general problem facing the industry with regard to frozen concentrate, with cost considerations taken into account, is the marketing of larger amounts of this product than have previously been experienced in the face of increasing competition. On a total basis within the price range specified by this study, a series of price decreases on the retail market will result in a greater proportion of sales offsetting the per unit revenue decrease, thus increasing total revenue. Within the various brand categories, the national and packer labels will gain in market share position at the expense of the private label. In terms of revenue, the net price effects are still elastic for each brand segment. However, the private brand's revenue curve is quickly approaching a maximum in that this net effect is very close to unity. As responses to price changes, while still absolutely increasing, begin to diminish in the lower price ranges, constant price alterations across all brands could jeopardize the position of the private label in this market. For this reason more demand information concerning the low extreme prices used in this study as well as prices beyond this range is needed. The role of disproportionate price decreases using simultaneous selection of the demand system can be utilized to assure that competiton within the category of frozen orange concentrate will not damage the market volume position of each brand.

The problem of short supplies and corresponding price increases is definitely a condition of loss of total revenue in the market studied. The elastic character of total demand is indicative of this phenomenon, with customers abstaning from consumption of concentrate or switching to other juice competitors at a greater percent rate than the rate of gain in per unit price. On a segmented basis, the packer brand loses market share position quickly, a condition that could possibly be attributed to lack of advertising. The national brand also decreases in market share position but not as drastically as the packer label. The private label, the most heavily advertised brand in the test region prior to the study, actually gained in market share position as prices rose for all three brands.

The dynamics of the marketplace in which frozen orange concentrate is sold require that marketing strategies be designed to maintain and improve the competitive position of concentrate as a single product on this market. However, these strategies should not ignore the inner workings of the demand system within frozen concentrated orange juice. The competition existing among brand classes is a very real conclusion reached in this study. The effects of price alterations are such that no brand group can be analyzed only in terms of a single price elasticity. Thus, the segments within this frozen product group must also be strengthened in terms of increased sales relative to the total volume of sales, but not in such a manner as to reap all gains from another concentrate label. Finally, implementation of pricing and promotional programs on the retail market for frozen orange concentrate should be supplemented by techniques of measurement that will feed back the results of these policies to the orange industry so that further alterations in planning may be made in preparation for an uncertain but important future.

CHAPTER VII

SUMMARY

The General Problem

In past years, the citrus industry in Florida has become more and more aware of the importance of consumer demand for all citrus products. Knowledge of consumer demand for frozen orange concentrate has been recognized as extremely important due to the large percentage of fruit that goes into the concentrate process. In recent years, technological advances in production techniques and synthesizing processes have brought into sharp focus the significance of marketing decisions pertaining to concentrate. Capacity of the industry to produce citrus fruit, even in light of the recent freeze, is of such magnitude as to indicate larger future supplies than previously realized in the citrus market. Moreover, due to the entrance of synthetic frozen concentrates into the juice market, the competition faced by orange products from Florida has increased and promises to become keener as time goes on. The importance of frozen orange concentrate as the largest single outlet for orange production in Florida makes it desirable that accurate information concerning the demand relationships for this product be obtained for the purpose of aiding in future decisions regarding production, pricing and promotional policies by the industry.

Objectives

In recognition of the need for demand information for frozen concentrated orange juice, this study was developed and carried out in an effort to accomplish the following objectives:

(1) Determination of a demand function for frozen orange concentrate.

(2) Measurement of the degree of economic substitution among three major classes of concentrate brands--one nationally advertised label, one packer label and one private label.

Research procedure

The framework within which the objectives were accomplished was the technique of controlled pricing. This methodology utilized data generated under controlled marketing conditions to evaluate the structural relationships within a postulated economic model. The model specified regarded the quantities of each brand of concentrate sold as functions of the prices of all three labels. A series of treatment combinations employing varying levels of these three brand prices were selected using a triple cube factorial design. Three alternative analytical models were established whereby standard multiple regression techniques utilizing these treatments as independent variables could be used in estimating the relationships between prices and quantities of concentrate sold.

General Conduct of the Study

Design components

A retail store design in which customers were exposed to the varying price combinations for the three brands of concentrate was developed in line with one consideration--that of reaching maximum attainable efficiency within practical limits with respect to approximation of price effects in the analytical models. The resulting experiment utilized 18 retail supermarkets over a period of six weeks. The price treatment combinations were assigned to these stores and time periods commensurate with the analytical techniques which were used in estimating the retail demand for concentrate.

Data generation

The complete experiment was conducted in 18 supermarkets located in the Dayton-Cincinnati area of Ohio beginning on May 11 and ending on June 20, 1964. Over 600,000 customers, who purchased three million dollars worth of items in these stores, were exposed to the test price combinations during this period. The sales of each brand of orange concentrate were tabulated on a daily basis in conjunction with the respective price level charged on a given day. All promotion and special merchandising of the test labels which could normally have been carried on by the cooperating agency was eliminated while the experiment was in effect, thus providing a retail atmosphere in which customer responses to varying prices could be recorded and analyzed.

The analytical process

The technique used for ascertaining the existence and nature of economic order in the sales data collected in the test stores, rather than being a separate statistical process, evolved from the development of the complete experimental design. The problems of heterogeneous store units and the possibility of short-run weekly alterations in sales across stores led to the conceptualization of a difference equation technique using changes in sales volume adjusted by an index of weekly variation as the dependent variable in the statistical equations. Moreover, consideration of the possible existence of a price-carryover effect in customer responses resulted in fitting one demand function per brand over all weeks and a similar function for each week. Within this framework, three equation forms were constructed for purposes of comparison in attaining the most useful estimates of the parameters in the economic model and were fit using a stepwise multiple regression technique.

Alternative analytical equation forms

The first analytical model used in analyzing the sales data was fit using differences between expected and actual concentrate sales in ounces per 300 customers as the dependent variables, and the base prices minus the actual prices charged as the independent variables. Expected sales in the test stores for each equation were calculated using information collected by design in a base week and two control stores which were integral components of the over-all experiment. These differences and the linear components of the price variables were converted to natural logarithms, completing the total specification of this model.

The equation form utilized in the second analytical model was a result of an algebraic expansion of arithmetic difference equations used in developing the preceding model. The final regression procedure contained the actual concentrate sales recorded in the test stores as the dependent variable, with the actual test prices charged as the independent variables. In addition, the square and cross-product terms of these prices were included in this regression. A final alteration of the price variables made the values used in regression conform to a special matrix designed to increase the efficiency with which estimates of certain parameters in the model could be obtained.

The third regression model was an expansion of the basic difference equation model from the first analytical technique to include square and cross-product terms of the actual differences in the base prices versus test prices. This process resulted in the dependent variable being arithmetic differences between base week sales and test week sales for a given store, with the independent variables constructed from the differences between the base and test prices.

Results of the Experiment

Each of the regression equations fit to the sales data was investigated through a combination of statistical and economic considerations with the results obtained in the first analytical model being chosen for the final analysis.

Segmented total demand

The price elasticity coefficients were estimated to be elastic for all three individual brand classifications. The most elastic was the packer brand with a coefficient of -4.37062, while the respective price elasticities for the private and national brands were -2.56857 and -2.28077. These values represented the average percentage change in quantity of each brand sold for a one percent change in the price of each label within the price range and area of this study.

Substitution effects

The nature of economic substitution among the three labels sold in the test market was ascertained by estimating cross-price elasticity coefficients relating the sales of each brand to the price levels of the other two brands. No significant relationship was found to exist between sales of the national brand and the packer label's price. However, a one percent change in the private label's price had a direct effect of .28769 percent on sales of the national brand. A one percent change in the national label's price had a direct effect on the packer brand's sales of .80669 percent, with the private label's price having a similar effect on packer sales of 1.17141 percent. The cross-price elasticity coefficients for the private label were .92884 for the national brand's price and .55157 for the packer price.

Net effects

The existence of measurable economic substitution in all but one case among the three brands of frozen orange concentrate precluded an analysis of the price effects in single dimensions. Rather, a more realistic evaluation of the demand structure was assumed to be embodied in net price effects for each brand. This concept was defined as the percentage in quantity sold for a given brand taking into account the substitution effects existing in the marketplace. For a constant percentage change in the prices of the three test brands, the net quantity change was calculated for the respective labels by adding the price elasticity and cross-price elasticity coefficients for each brand. When disproportionate price changes were used, the net effects were altered from a single sum to a weighted sum based on the varying percentage changes for each price.

Total demand

A total demand curve, made up of a composite of the individual demand relations established in this study, was assumed to exist in the test market. Approximation of the price elasticity of this curve was done by using the average prices charged and quantities sold for each brand, and a constant price deviation of plus or minus 10 percent from this point. An arc elasticity formula, utilizing the resulting quantities, was used in deriving a total price elasticity estimate of -1.7542 for a 10 percent price increase and -1.4923 for a 10 percent price decrease.

Implications

The determination of marketing policy to deal with general anticipated market conditions for frozen orange concentrate is a continuing operation in the Florida orange industry. A major component of this policy is the development of short-run strategies designed to cope with special situations which occur in the dynamic market for fresh and processed juices. In most instances, these special conditions call for an adjustment in price since promotion and supply changes usually experience a lagged effect after their inception. The market examined in this study revealed certain characteristics of consumer responses to varying prices for frozen orange concentrate that should be helpful in the formulation of these special strategies.

In terms of a composite demand for the three brand classifications of concentrate, the response to price changes was relatively elastic. However, the lowering of prices resulted in slightly less response than an increase in price; indicating that on a total basis, the revenue curve of the industry is approaching a maximum at a diminishing rate for lower price levels than those in evidence in the test market. While the individual price elasticities of demand for each brand were found to be very elastic, consideration of single labels was deemed secondary to analysis of the whole system of demand functions. The existence of economic interaction among the three brands and their respective prices was found to be such that net price effects were more meaningful for interpretation than single price effects.

The pluralistic nature of price effects on the market for frozen orange concentrate is shown in this study. Even though the magnitude of these effects will vary among market regions, the existence of these relationships and the competitive tendencies they suggest are very real considerations which should be taken into account by pricing strategists in the orange industry of Florida.

APPENDIXES

APPENDIX I

METHODOLOGY CRITIQUE

METHODOLOGY CRITIQUE

A series of assumptions were made during the progress of this study which were critical to the final analysis of the results. Of singular importance were the assertions made concerning period and store variability in the data and the basic regression procedure used in the analytical process. This section is presented as a brief clarification and discussion of these factors for the possible benefit of future research.

Period division within weeks

The reduction in size of the complete experiment stipulated in Chapter III was accomplished by dividing each week into 3-day periods. Each period was regarded as a basic time unit within each supermarket. The assumption underlying this technique was that customers shopping for frozen orange concentrate in a given store during a given period would continue shopping for concentrate in the same period from week to week. In this manner, two price treatments could be allocated to a single store design with the 3-day periods regarded as homogeneous units.

Examination of customer counts and dollar purchase volumes, coupled with recognition of a large degree of variability in the concentrate sales data, resulted in the addition of a period variable within weeks to each regression model. The statistical significance of this new variable tended to invalidate the homogeneity assumption made earlier for periods within weeks. However, each period response was compared to the proper corresponding base period value in the difference equation analysis. No assumption was made concerning homogeneity of sales responses among periods in that each unit received a unique test price combination. The extent to which period effects and price effects interacted with one another could not be ascertained for this design in that this was not a consideration in the price allocation process. The calculation of a coefficient and sum of squares for periods was regarded as effectively compensating for the average period influence, thus allowing estimation of price effects free of period disturbances.

Store traffic variability

The problem of compensating for traffic volume differences among the test supermarkets was resolved by expressing the dependent variable in the regression procedure as differences in average purchase rates per 300 customers. It was assumed that after store, week and period effects were removed from total variability that purchase rates would tend to be the same across these factors if prices were held constant. The question of interaction between traffic volume and purchase rates either between days or across periods was not considered. It appeared that if an interaction did exist, it would be consistent across periods, thus allowing examination of percentage changes in purchase rates between units in the light of corresponding price changes.

The consistency in differences in purchase rates adhered to above was also considered important in guiding the algebraic calculation of the dependent variable for regression. This calculation assigned equal weight to the purchase rates on each day within a given period as well as between periods. This procedure was based on the assumption that the differences in purchase rates would be consistent between days when test period responses were compared with base period purchase rates. Therefore, purchases in ounces sold per 100 customers were aggregated over the 3-day period. An interaction between customer traffic volume and purchase rates plus a lack of consistency in the percentage responses in quantity sold would result in this aggregation procedure giving more weight to customer responses on slow traffic days as compared to heavy shopping days.

APPENDIX II

INPUT DATA

Table 32.--Quantity of nationally advertised, packer and private brands sold in ounces per 300 customers by period and price combination, Dayton-Cincinnati, Ohio, May-June 1964

Observation number	Code designation*	Price per brand			Quantity sold in ounces per 300 customers		
		Nationally advertised	Packer	Private	Nationally advertised	Packer	Private
001	0111	29	25	25	4.04949	8.52479	37.35595
002	0121	29	25	25	11.24301	15.98945	27.25110
003	0112	29	17	25	5.01364	26.26675	19.79648
004	0122	25	21	21	7.52633	17.01423	42.56384
005	0113	29	17	25	1.22449	18.25660	11.46370
006	0123	25	21	21	9.19825	14.01747	72.32286
007	0114	29	17	25	7.01532	15.09840	19.67496
008	0124	25	21	21	13.27909	16.34054	45.75610
009	0115	29	17	25	3.76771	25.00665	12.69004
010	0125	25	21	21	2.81897	11.87337	48.89858
011	0211	29	25	25	18.69866	11.10282	73.15019
012	0221	29	25	25	15.48169	60.44632	107.19835
013	0212	29	25	17	10.41427	15.23071	112.88741
014	0222	33	21	21	18.33419	51.81341	107.90816
015	0213	29	25	17	9.46283	14.52084	89.22261
016	0223	33	21	21	17.96127	67.49763	126.29303
017	0214	29	25	17	13.71327	10.79263	93.85974
018	0224	33	21	21	15.56321	58.93260	115.93118
019	0215	29	25	17	9.59919	2.91616	104.75823
020	0225	33	21	21	12.07467	49.98813	111.00685
021	0311	29	25	25	21.74491	26.13747	98.45144
022	0321	29	25	25	39.04060	74.34808	178.39558
023	0312	29	25	25	16.20622	32.46347	102.08059
024	0322	29	25	25	34.07870	48.19677	137.97119
025	0313	29	25	25	22.47613	28.00969	87.46751

Table 32.--Continued

Observation number	Code designation*	Price per brand			Quantity sold in ounces per 300 customers		
		Nationally advertised	Packer	Private	Nationally advertised	Packer	Private
026	0323	29	25	25	31.74586	57.67741	181.39900
027	0314	29	25	25	24.25399	19.60817	94.71322
028	0324	29	25	25	30.45250	44.58862	183.76015
029	0315	29	25	25	23.07690	29.00400	87.09855
030	0325	29	25	25	34.96926	48.37710	159.20542
031	0411	29	25	25	45.90482	44.65599	129.49069
032	0421	29	25	25	49.52941	72.09339	234.36402
033	0412	29	25	25	32.07300	33.89545	100.06941
034	0422	27	27	27	90.83072	54.61785	219.77139
035	0413	29	25	25	33.42974	43.85139	144.29428
036	0423	27	27	27	73.16307	66.03983	208.28831
037	0414	29	25	25	40.00479	22.32286	100.63790
038	0424	27	27	27	69.25783	47.84518	119.93043
039	0415	29	25	25	47.36485	28.85348	134.84433
040	0425	27	27	27	66.42640	47.58610	136.82404
041	0511	29	25	25	20.17324	20.52963	71.50441
042	0521	29	25	25	42.27770	54.94946	162.00879
043	0512	31	23	27	31.81728	34.24484	55.67386
044	0522	35	19	31	22.95468	106.09394	62.07620
045	0513	31	23	27	21.78972	67.83663	52.75861
046	0523	35	19	31	33.13548	119.11679	91.67556
047	0514	31	23	27	15.96326	41.35959	53.81557
048	0524	35	19	31	20.62034	85.12924	62.22273
049	0515	31	23	27	20.13742	31.23709	51.83790
050	0525	35	19	31	27.40435	102.04544	65.89165
051	0611	29	25	25	30.17695	27.65487	46.35362
052	0621	29	25	25	22.93895	38.38545	108.93572

Table 32.--Continued

Observation number	Code designation*	Price per brand			Quantity sold in ounces per 300 customers		
		Nationally advertised	Packer	Private	Nationally advertised	Packer	Private
053	0612	29	25	33	25.27101	25.60664	8.33252
054	0622	33	21	29	13.97413	83.46643	58.42216
055	0613	29	25	33	18.73306	16.29542	14.86699
056	0623	33	21	29	15.80816	55.30042	57.38876
057	0614	29	25	33	28.98033	32.23676	17.64661
058	0624	33	21	29	12.33693	55.61156	64.12874
059	0615	29	25	33	13.12813	36.88277	23.64328
060	0625	33	21	29	19.47413	53.03340	60.95000
061	0711	29	25	25	14.03973	21.60759	64.14824
062	0721	29	25	25	26.76407	72.04482	129.65278
063	0712	21	25	25	45.75805	21.26799	27.44032
064	0722	25	21	29	42.01962	82.64832	70.62891
065	0713	21	25	25	28.48665	5.26316	36.44646
066	0723	25	21	29	37.98800	77.22536	82.06551
067	0714	21	25	25	49.92095	3.83700	60.62227
068	0724	25	21	29	54.89710	101.82607	62.47014
069	0715	21	25	25	36.13319	20.46482	44.44614
070	0725	25	21	29	37.80420	93.22203	68.77958
071	0811	29	25	25	7.23506	33.48521	76.76548
072	0821	29	25	25	34.77925	61.13108	190.43298
073	0812	29	25	25	16.07695	29.90540	100.88963
074	0822	29	25	25	33.80422	47.14492	151.84489
075	0813	29	25	25	17.47613	44.83294	98.53764
076	0823	29	25	25	47.15000	43.94194	174.28901
077	0814	29	25	25	16.80813	34.24071	73.34307
078	0824	29	25	25	35.05316	34.63094	192.76828
079	0815	28	25	25	46.81045	30.72552	84.37098

Table 32.--Continued

Observation number	Code designation*	Price per brand			Quantity sold in ounces per 300 customers		
		Nationally advertised	Packer	Private	Nationally advertised	Packer	Private
080	0825	29	25	25	38.22858	36.68677	149.08815
081	0911	29	25	25	5.82524	31.79978	16.88233
082	0921	29	25	25	11.65370	23.30826	74.48547
083	0912	27	23	27	2.69934	16.50666	10.51385
084	0922	23	19	31	20.78176	62.74659	20.00834
085	0913	27	23	27	0.00000	12.40891	28.63054
086	0923	23	19	31	14.06371	84.92166	25.80250
087	0914	27	23	27	19.07056	12.23985	13.24020
088	0924	23	19	31	12.00775	55.87520	17.39346
089	0915	27	23	27	12.80982	19.64231	14.65517
090	0925	23	19	31	9.36810	65.30320	27.25906
091	1011	29	25	25	1.61290	14.67075	18.03785
092	1021	29	25	25	3.56471	24.21165	43.00283
093	1012	35	31	19	0.00000	3.20929	54.81467
094	1022	31	27	23	2.64221	6.03723	54.85444
095	1013	35	31	19	1.38013	0.00000	46.32609
096	1023	31	27	23	1.80332	15.23555	49.48916
097	1014	35	31	19	1.33185	0.00000	50.45971
098	1024	31	27	23	2.04511	14.66406	43.63203
099	1015	35	31	19	0.00000	1.37773	117.64330
100	1025	31	27	23	2.23755	10.99264	49.84916
101	1111	29	25	25	62.21229	31.90323	129.74799
102	1121	29	25	25	30.37169	46.50936	77.26037
103	1112	23	31	31	82.15894	3.13553	17.55866
104	1122	25	29	29	30.46757	9.37706	36.90176
105	1113	23	31	31	45.53116	12.73715	22.34258
106	1123	25	29	29	114.43385	13.66320	73.01654

Table 32.--Continued

Observation number	Code designation*	Price per brand			Quantity sold in ounces per 300 customers		
		Nationally advertised	Packer	Private	Nationally advertised	Packer	Private
107	1114	23	31	31	45.07637	1.47965	29.77367
108	1124	25	29	29	78,78166	14.04099	32.48172
109	1115	23	31	31	79.07651	11.17990	26.36109
110	1125	25	29	29	69.35703	22.76339	31.72588
111	1211	29	25	25	27.42486	49.96997	69.69027
112	1221	29	25	25	49.11592	65.94220	124.22800
113	1212	29	25	25	10.22113	36.38771	72.41142
114	1222	27	27	23	43.49176	30.92486	97.99064
115	1213	29	25	25	13.70943	39.82944	70.65803
116	1223	27	27	23	46.97901	43.62565	167.99368
117	1214	29	25	25	33.37491	30.30954	86.46368
118	1224	27	27	23	29.13338	28.83446	109.35181
119	1215	29	25	25	36.09869	55.21141	85.32251
120	1225	27	27	23	30.25002	33.53615	110.08200
121	1311	29	25	25	25.92070	7.08834	82.39145
122	1321	29	25	25	27.26496	40.22522	112.40353
123	1312	35	19	19	8.76221	10.07850	115.19589
124	1322	31	23	23	11.16630	22.20046	62.35221
125	1313	35	19	19	25.11595	11.27170	138.34633
126	1323	31	23	23	29.95416	39.42718	130.50644
127	1314	35	19	19	9.28960	17.45932	61.50795
128	1324	31	23	23	17.80093	29.30745	112.88488
129	1315	35	19	19	2.89895	23.07612	76.26113
130	1325	31	23	23	15.77641	33.51603	106.66283
131	1411	29	25	25	22.43350	44.51360	158.58136
132	1421	29	25	25	57.34198	94.40256	219.73125
133	1412	23	19	19	35.82974	64.45195	146.53674

Table 32.--Continued

Observation number	Code designation*	Price per brand			Quantity sold in ounces per 300 customers		
		Nationally advertised	Packer	Private	Nationally advertised	Packer	Private
134	1422	27	23	23	49.01270	96.48182	227.92246
135	1413	23	19	19	47.75589	48.84229	122.07415
136	1423	27	23	23	71.05043	73.34008	225.18137
137	1414	23	19	19	34.35747	36.50088	139.09082
138	1424	27	23	23	51.71713	65.20977	208.67783
139	1415	23	19	19	44.41140	77.60076	194.21534
140	1425	27	23	23	58.69838	72.88918	239.04860
141	1511	29	25	25	11.32482	7.17160	49.71391
142	1521	29	25	25	12.38335	13.26599	60.00149
143	1512	33	29	21	10.13265	.68807	29.79291
144	1522	29	33	25	12.99975	2.05695	73.21113
145	1513	33	29	21	6.56384	1.13101	36.50057
146	1523	29	33	25	12.99082	4.25865	77.27252
147	1514	33	29	21	1.96385	0.00000	31.95694
148	1524	29	33	25	13.64783	1.60915	54.90102
149	1515	33	29	21	8.89764	2.73660	71.59761
150	1525	29	33	25	13.35647	1.34228	72.93032
151	1611	29	25	25	16.01251	21.82310	52.57358
152	1621	29	25	25	17.75612	36.06529	96.79344
153	1612	25	29	21	13.06207	4.21699	76.14216
154	1622	23	31	19	23.43521	6.75472	100.51695
155	1613	25	29	21	16.07090	10.87137	48.04650
156	1623	23	31	19	32.24508	6.55094	135.80191
157	1614	25	29	21	14.24423	11.12384	89.02183
158	1624	23	31	19	23.98227	3.57745	120.31950
159	1615	25	29	21	15.41867	4.05026	64.17376
160	1625	23	31	19	25.06623	5.49845	97.15529

Table 32.--Continued

Observation number	Code designation*	Price per brand			Quantity sold in ounces per 300 customers		
		Nationally advertised	Packer	Private	Nationally advertised	Packer	Private
161	1711	29	25	25	41.43166	41.14786	80.24774
162	1721	29	25	25	44.12453	80.53132	132.45665
163	1712	31	27	27	39.74632	39.22444	73.04023
164	1722	35	31	31	19.99317	28.76998	51.92712
165	1713	31	27	27	32.12643	53.23107	62.91064
166	1723	35	31	31	21.71849	36.58478	70.82627
167	1714	31	27	27	39.98749	51.71421	63.95243
168	1724	35	31	31	27.95094	24.86994	63.18575
169	1715	31	27	27	44.24767	52.02184	108.15301
170	1725	35	31	31	30.85191	29.36261	58.94326
171	1811	29	25	25	25.23254	25.10263	59.64733
172	1821	29	25	25	44.78368	68.17916	110.31141
173	1812	37	25	25	21.76711	41.69778	70.60783
174	1822	33	29	29	32.38325	42.50890	68.98730
175	1813	37	25	25	21.70686	27.42111	105.15071
176	1823	33	29	29	23.27067	49.04498	92.24017
177	1814	37	25	25	10.64576	16.03146	70.16229
178	1824	33	29	29	26.86981	33.85225	89.84767
179	1815	37	25	25	24.68974	31.54311	77.70540
180	1825	33	29	29	24.58914	29.91960	70.67008

*The first two columns in the code designation indicate the store number, with stores being ranked consecutively as 1-18, while the last two columns refer to week and period number in that order.

Table 33.--Average sales of the nationally advertised brand in ounces per 300 customers by week and price level, 16 test stores, Cincinnati-Dayton, Ohio, May-June 1964

Price brand 1	Ounces per 300 customers				
	Week 2	Week 3	Week 4	Week 5	Total average
21	41.07611	20.66322	35.23217	14.98324	27.98868
23	38.50006	27.74913	24.15479	21.48332	27.97183
25	24.68423	40.71976	43.91580	29.32709	34.66172
27	50.44813	44.72289	45.65495	40.50173	45.33192
29	14.77299	11.28144	17.04183	9.84436	13.23515
31	19.81435	17.20579	15.46318	11.21635	15.92492
33	19.86132	14.53286	15.77688	15.07613	16.31180
35	13.64241	17.63602	15.55807	14.98835	15.45622
37	19.53991	15.74540	7.51334	10.23802	13.25917
Total average	25.53300	23.57384	24.59659	19.20810	23.22789

Table 34.--Average sales of the packer brand in ounces per 300 customers by week and price level, 16 test stores, Cincinnati-Dayton, Ohio, May-June 1964

Price brand 2	Ounces per 300 customers				
	Week 2	Week 3	Week 4	Week 5	Total average
17	25.11019	14.94327	16.71728	24.91692	20.43316
19	77.79218	80.30717	75.22198	91.75722	81.26964
21	83.46243	71.33995	99.49392	82.86569	84.29050
23	54.29066	54.00591	55.24682	55.08435	54.65193
25	27.73683	20.07832	21.31961	29.25953	24.59857
27	41.90777	52.52217	53.36817	49.65948	49.36440
29	19.60457	23.35670	23.55557	22.67041	22.29681
31	14.13638	16.98354	12.57206	17.01438	15.17659
33	2.92290	5.67764	2.75193	2.13782	3.37257
Total average	42.47593	41.72351	44.53815	46.21056	43.73739

Table 35.--Average sales of the private brand in ounces per 300 customers by week and price level, 16 test stores, Cincinnati-Dayton, Ohio, May-June 1964

Price brand 3	Ounces per 300 customers				
	Week 2	Week 3	Week 4	Week 5	Total average
17	97.45166	84.04773	97.85896	107.04767	96.60150
19	100.29620	107.44359	94.90368	128.20854	107.71300
21	70.73627	71.39923	71.12858	82.51056	73.94366
23	140.98184	148.58647	116.21056	151.23201	139.25272
25	57.29970	71.13264	67.62040	75.00292	67.76391
27	99.96952	87.97844	63.51704	85.53825	84.25081
29	74.74795	78.99206	69.95944	69.42614	71.03140
31	46.42625	54.07694	42.73099	52.22389	48.86452
33	7.19317	14.00471	18.39851	24.15999	15.93909
Total average	80.65859	84.96110	72.49315	89.30571	81.85464

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BIOGRAPHICAL SKETCH

Kenneth Robert Henderson was born on March 9, 1938 at Chicago, Illinois. He attended public schools in Wheaton and Peru, Illinois and graduated from Bethesda Chevy Chase Senior High School, Bethesda, Maryland in 1956.

The author received his Bachelor of Science degree in Agriculture from the University of Florida in January 1961. He subsequently enrolled in the graduate school of the University of Florida in February 1961 and has been working towards the degree of Doctor of Philosophy in Agricultural Economics since that time.

The writer is a member of Florida Blue Key, Gamma Sigma Delta, Alpha Zeta, and Scabbard and Blade Military Honor Society. He holds the rank of First Lieutenant in the United States Army Reserve.

The author is married to the former Susanne Garrison, and they have two children, Gregory Bruce and James Douglas.

This dissertation was prepared under the direction of the chairman of the candidate's supervisory committee and has been approved by all members of that committee. It was submitted to the Dean of the College of Agriculture and to the Graduate Council, and was approved as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

June 22, 1965

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